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# Analysis of the Pseudorapidity Distributions of the Shower Particles in Relativistic Heavy Ion Collisions

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ABSTRACT

**The pseudorapidity distributions of shower particles produced in collisions of $^{16}O$ nuclei with emulsion targets at 3.7 A GeV at JINR Dubna are analyzed and compared with similar distributions from interactions involving $^{12}C$ and $^{28}Si$ nuclei at the same energy, as well as $^{22}Ne$ at 3.3 A GeV. The resulting distributions are suitably fitted with Gaussian functions. The width (dispersion) of these distributions shows only a weak dependence on the type of projectile, while the mean pseudorapidity increases with increasing projectile mass. Furthermore, higher-order statistical moments of the distributions such as skewness and kurtosis are evaluated and discussed with characterize their shape and symmetry. It is also observed that the total charge of projectile fragments that did not undergo interaction, along with the effective size of the target nucleus, significantly influences the resulting pseudorapidity patterns. These findings help clarify the underlying mechanisms of particle production in nucleus–nucleus collisions at intermediate energies (Dubna energy).**

## 1- INTRODUCTION

In the center of mass frame, the two colliding nuclei are Lorentz contracted and overlap according to the interaction impact parameter b, which is the distance between the centers of the colliding nuclei in the plane perpendicular to the direction of incidence. Therefore, the nucleons in the overlap region are considered as participant nucleons from both projectile and target in the interaction, while the nucleons outside the overlap region are regarded as spectators. The spectator nucleons from the projectile continue their motion in the forward direction without suffering interactions. In the overlap region of both colliding nuclei a hot and dense hadronic matter is formed, whose evolution is described by fireball models [1-4]. If the fireball temperature or density becomes larger than a critical value, quark – gluon plasma (QGP) can be created. After short time, the formed hadronic matter fireball begins to expand, cooling gradually, until hadronization occurs leading to the emission of large number of hadrons, mainly pions. Nearly the total participant nucleon energy is spent in pion creation [4]. The rapidity distributions of the created pions can be treated in the light of the fireball models [1-2]. According to those models and the string model [3], the incident nucleons penetrate through the target where a string is formed along the direction of incidence. In nucleus–nucleus collisions the length of the string is defined as the width of the pseudorapidity distributions (PRD). Under the assumption that the particles are emitted isotropically in fireball rest frame, the PRDs of the emitted relativistic particles are approximately a Gaussian shape. [1-4]

At JINR-Dubna energy, EMU01 Collaboration and others [5-11] found that relativistic charged pions (shower particles) constitute more than 90% of the produced particles, with contamination of not more than 10% of protons, baryons and K-mesons. In the 3.7A GeV $^{16}O$ interactions with emulsion nuclei discussed in this paper, the shower particles are regarded as charged pions. The emulsion is used as a target with wide range

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of nuclei with mass number $A_T$ from 1 (H) up to 108 (Ag). In emulsion experiments, the impact parameter of the colliding nuclei cannot be measured. Consequently, it is essential to establish selection criteria for central (small impact parameter) and quasi-central collisions in order to explore the extreme states of nuclear matter. The nuclear emulsion detector gives the full picture of the interaction event. It is used for accurate measurements of space emission angles of all charged particles emerging from the collision relative to projectile direction with high resolution ( ≈ 0.1 mrad). This advantage allows for a successful examination of the PRDs of the emitted shower particles. [see Eqn. (2)].

In previous papers [12-15], the authors studied the multiplicity characteristics of different produced particles (shower, grey and black) in $^{16}$O- Emulsion ($^{16}$O-Em) interactions at 3.7A GeV. In this work, the angular characteristics (pseudorapidity) of the emitted shower particles are studied. Details on the experiment can be found in Ref's [14]. This paper seeks to investigate the production of shower particles resulting from the inelastic collisions of $^{16}$O nuclei at an energy of 3.7A GeV with emulsion nuclei, focusing on various centrality levels of the collisions. A systematic comparison is made with the data from the interactions of $^{12}$C and $^{28}$Si at energy 3.7A GeV in addition to the data for 3.3A GeV $^{22}$Ne with the same type of emulsion [16-18]. The various order moments of the pseudorapidity distributions of shower particles are calculated, in the standard way, for 3.7A GeV $^{16}$O-Em interactions in different centrality classes to draw the final conclusions.

## 2- RAPIDITY AND PSEUDORAPIDITY

Within high-energy nuclear collision scenarios, the parameter known as rapidity, y, is a convenient relativistic invariant that describes the motion along the direction of the incident beam. The rapidity parameter is crucial for understanding the dynamics involved in the production of hadron particles.

The rapidity y is given by (c=1),

$$y = \tfrac{1}{2} \ln \left(\frac{E+P_L}{E-P_L}\right) \qquad (1)$$

where E and $P_L$ are the total energy and the longitudinal momentum of the emitted particle respectively. For relativistic pions $E \approx P$, $P_L = P \cos\theta$ and $E >> m_\pi$. where θ is the angle of the emitted particle with respect to incident projectile direction, and $m_\pi$ is the mass of the pion ≈ 139 MeV. Therefore, the rapidity can be approximated by the pseudorapidity η,

$$y \approx \eta = \tfrac{1}{2} \ln \left(\frac{P+P_L}{P-P_L}\right) = \tfrac{1}{2} \ln \left[\frac{1+\cos\theta}{1-\cos\theta}\right] = -\ln \tan\frac{\theta}{2} \qquad (2)$$

In case of pions the difference between y and η is negligible, since the mean transverse momentum, at JINR energy is typically $<P_t> \approx 350$ MeV, large compared to their mass $m_\pi = 139$MeV, so that $\eta - y \approx 0.07$. Anyway, since no accurate momentum measurements are provided in emulsions experiments, η is used as convenient variable to describe the produced particles.

Pseudorapidity, is a crucial parameter for analyzing the mechanisms of particle production in high-energy collisions between nuclei. It serves as an approximation of the dimensionless boost parameter of particle rapidity, particularly relevant in the framework of Lorentz boosts. From the study of the PRDs (dN/dη), the maximum pseudorapidity $\overline{\eta}$, the values of the different moments of each distribution and the dependence of these parameters on the collision geometry are our core interest in this paper. The different η-distributions can be described by Gaussian distributions [16 – 18].

## 3- EXPERIMENTAL DETAILS

The NIKFI-BR2 nuclear emulsion stacks have been exposed to a 3.7A GeV $^{16}$O beam at the synchrophasotron located in Dubna, Russia, under the auspices of JINR. Each pellicle within the stack measures 20x10x0.06 $cm^3$. Following the development process, the emulsion stacks exhibit a shrinkage factor of approximately 2.2 in thickness. The sensitivity of the developed emulsion plates is assessed by measuring the grain density (i.e. the number of particle track ionization center per 100μm seen as small black spots (grains) in the emulsion) associated with minimum ionizing particle tracks. The atomic density (ρ), of each emulsion element is listed in Table (1).

**Table (1): Atomic density of nuclei for NIKFI – BR2 emulsion**

| Element | $^{1}$H | $^{12}$C | $^{14}$N | $^{16}$O | $^{80}$Br | $^{108}$Ag |
|---|---|---|---|---|---|---|
| $\rho \times 10^{22}$ $cm^{-3}$ | 3.150 | 1.410 | 0.395 | 0.956 | 1.028 | 1.028 |

The optical instruments and equipment utilized in the scanning technique and particle identification have been extensively discussed in earlier publications [15]. The charged secondary tracks are identified in the emulsion, according to the following the common terminology [19,20]:

i) Relativistic shower particles (s-particles) of relative ionization $I/I_o < 1.4$, where I is the particle track ionization emitted from the interaction vertices and $I_o$ is the ionization at the plateau of minimum ionizing single charged particles. These particles practically consist of pions produced by the interactions. Tracks of this type with an emission angle $\theta \leq 3^o$ were further subjected to multiple coulomb-scattering measurement for momentum determination and consequently for separating produced pions from singly charged projectile fragments (PFs) (protons, deuteron or tritons). The shower created particles are mostly charged pions with speed $\beta = \upsilon/c \geq 0.7$, having K.E > 70MeV. Their multiplicity is denoted as $n_s$.

ii) Grey particles (g-particles) with range L > 3mm and $1.4 \leq I/I_o < 4.5$ with $\theta > 3^o$ are mainly knockout protons from the target nucleus. The multiplicity of these tracks is denoted by $N_g$. Their energy lies in the range 26 < K.E < 400 MeV. But grey particles with $\theta \leq 3^o$ with $I/I_o \approx 4$ and without change in ionization along a length at least 2 cm from the interaction vertex are taken as doubly charged Q = 2, PFs.

iii) Black-particle tracks, known as b-particles, are defined by a length of L ≤ 3 mm and an intensity ratio of $I/I_o \geq 4.5$. The multiplicity of these particles is denoted as $N_b$. They are mainly evaporated target protons, with kinetic energies ≤ 26 MeV. In each event, the grey plus black track particles, $N_h = N_g + N_b$, are known as heavily ionizing target fragments.

iv) The PFs are those isotopes fragmented from the incident projectile emitted in the narrow forward cone with $\theta \leq 3^o$. The charge of PFs was determined using a combined technique of grain gap and δ-ray densities. More details of the charge determination of the emitted PFs have previously been published [21].

In each $^{16}$O-emulsion ($^{16}$O-Em) interaction event, the total charge of the projectile spectator $Q = \sum_1^8 N_i Z_i$ , was estimated, where $N_i$ is the number of PFs having charge $Z_i$ ($N_i$ takes a value from 1 to 8 in our $^{16}$O projectile). Thus, the number of the stripped nucleons, Q can be used as a selection criterion for the degree of collision centrality in each event [21, 22]. Events with Q = 0 are considered as pure central collisions according to Herkmann et.al. [22] and the degree of centrality decreases as the value of Q increases.

The emission angle with respect to incident projectile direction, θ, for the emitted shower particles is measured using the special KSM1 microscope of magnification 750X. More details about the procedure of angular measurement can be found in Ref. [23].

Other authors [24] suggested that events with $N_h \geq 28$ were chosen to represent pure central collisions for interactions of different projectiles with AgBr in emulsion. It is known that emulsion is a target composed of hydrogen (H), CNO (the light group) and AgBr (the heavy group). The effective masses of these groups are 1,14 and 94 respectively. We considered here after the two above definitions for pure centrality.

## 4- RESULTS AND DISCUSSION

### 4.1 Projectile size dependence

Figures [1(a) – 1(d)] illustrate the PRDs, dN/dη, of emitted shower particles resulting from the interactions of $^{12}$C-Em, $^{16}$O-Em and $^{28}$Si-Em at incident energy 3.7A GeV as well as those emitted from $^{22}$Ne-Em interactions at 3.3A GeV. Each distribution is normalized to same total number of shower particles (100%). For convenient comparison the experimental data, denoted by closed squares in the figures, for each projectile fitted using a single Gaussian distribution [17, 18].

$$dN/d\eta = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{(\eta - \overline{\eta})^2}{2\,\sigma^2}\right] \qquad (3)$$

where $\sigma^2$ is the variance and $\overline{\eta}$ is the average pseudorapidity. The values of $\sigma^2$ and $\overline{\eta}$ are used as free parameters to fit the experimental data in the interval of $0 \leq \eta \leq y_p$ , where $y_p$ is the projectile rapidity, ($y_p \approx 3.0$ at 3.7 GeV). The fitting has been performed on the positive η values as suggested by the KLM Collaboration [18], since the negative values are due to different emission mechanism [5, 16] .The deviation from the Gaussian fits at the tails of the distributions may be due to small relativistic proton contamination emitted in the forward direction near the projectile pseudorapidity. This hypothesis has been previously supported by Refs. [11, 16]. In the following discussion, the values of $\sigma^2$ and $\overline{\eta}$ are calculated as defined by statistics, directly from the experimental data of the PRDs for each projectile and listed in table (2).

**Table (2): Values of M (= $\overline{\eta}$), variance ($\sigma^2$) skewness (S), kurtosis, Excess (K-3), scaled variance ($\omega$) and the maximum pseudorapidity density $\rho_{max}$ of the PRDs of shower particles in $^{12}C$ – Em , $^{16}O$ – Em and $^{28}Si$ – Em interactions at 3.7A GeV and $^{22}Ne$ – Em interactions at 3.3A GeV.**

| Interaction | No. of events | M (= $\overline{\eta}$), | $\sigma^2$ | S | K | Excess K-3 | $\omega = \left(\frac{\sigma^2}{M}\right)$ | $\rho_{max}$ | Ref. |
|---|---|---|---|---|---|---|---|---|---|
| $^{12}C$–Em | 819 | 1.59±0.19 | 1.52±0.02 | -0.24±0.03 | 0.39±0.06 | - 2.61 | 0.95 | 2.31 | 28 |
| $^{16}O$–Em | 1875 | 1.72±0.01 | 1.96±0.03 | -0.18±0.02 | 0.41±0.03 | - 2.59 | 1.13 | 2.18 | Present work |
| $^{22}Ne$ –Em | 3812 | 1.95±0.01 | 1.73±0.02 | -0.34±0.01 | 0.67±0.02 | - 2.33 | 1.29 | 2.55 | 28 |
| $^{28}Si$ – Em | 1209 | 1.80±0.02 | 2.19±0.02 | -0.06±0.02 | 1.51±0.04 | - 1.49 | 1.23 | 3.14 | 28 |

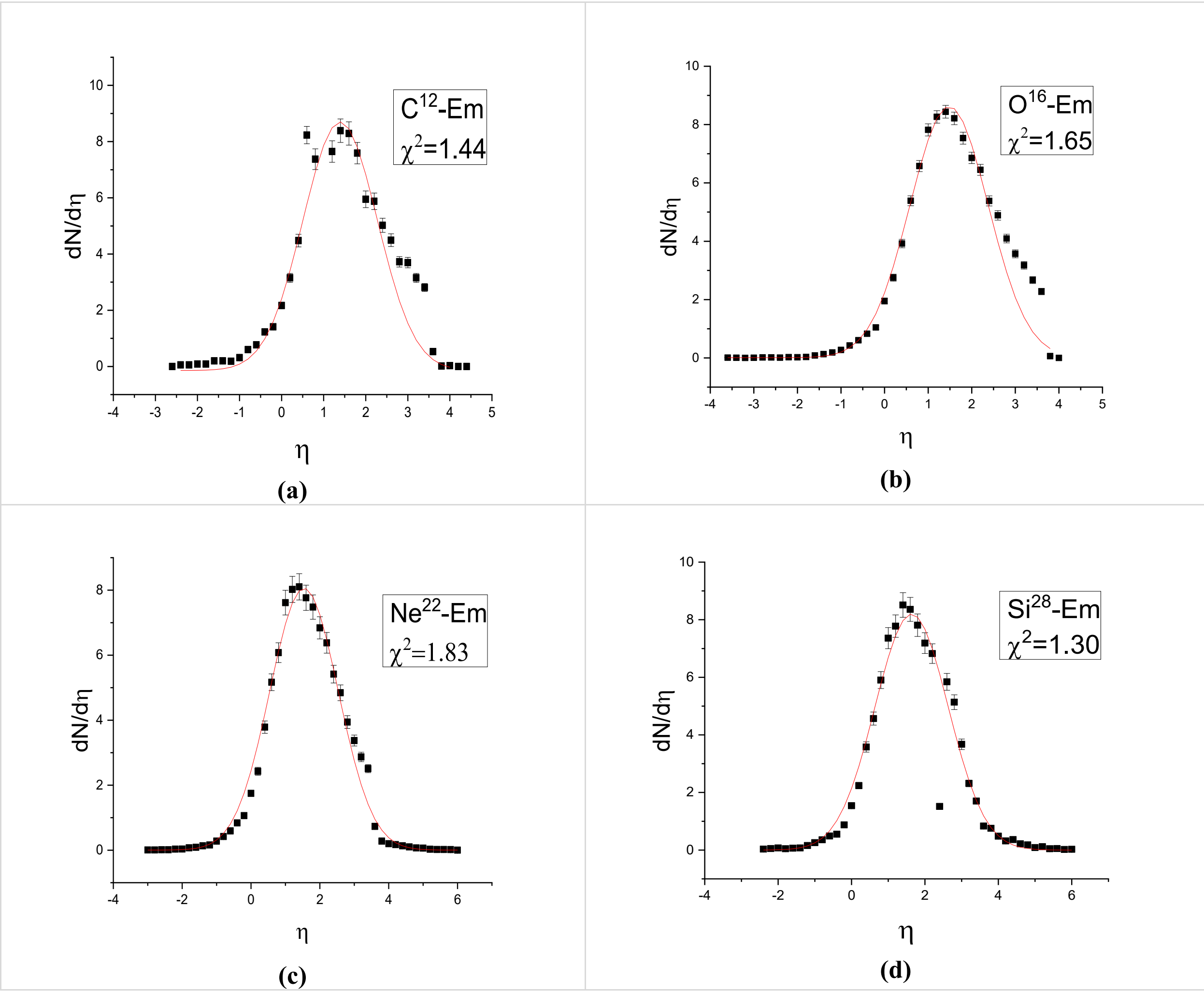


**Fig. (1): (a)-(b). The PRDs of pion production for $^{12}C$, $^{16}O$ and $^{28}Si$ interactions with emulsion nuclei at 3.7AGeV as well as $^{22}Ne$ interaction with emulsion nuclei 3.3AGeV fitted by Gaussian distribution (smooth curve). Only statistical errors are shown. All distributions are normalized to the same number of particles (100%).**

The shapes of the PRDs distributions are characterized by their statistical moments [41]. Those moments such as mean M (= $\overline{\eta}$ ), variance $\sigma^2$, kurtosis (K) and skewness (S), in addition to the scaled variance $\omega = \left(\frac{\sigma^2}{M}\right)$ are mostly used to characterize the properties of the probability distributions. S is used to describe the asymmetry, being the third moment normalized to $\sigma^3$ and K (defined by normalized fourth order moment) describes the peakness of the distribution. More details on their various moments were discussed in Refs. [25, 26]. If we assume that Gaussian distribution can be used to describe the PRDs it is possible to relate the maximum shower particle density $\rho_{max}$ to the total shower multicity $\langle n_s \rangle$ as follow $\rho_{max} = \langle n_s \rangle / \left(\sqrt{2\pi}\ \sigma\right)$ where σ is the dispersion of the PRDs ($\sqrt{\sigma^2}$ ) [11]. Note that the values of $\rho_{max}$ for different projectile interactions, which are shown in table (2) are partially independent on the projectile mass number at 3.7A GeV.

From Figs. [1(a) – 1(d)] and Table (2) we notice that:

1. The PRDs of shower particles emitted from the interactions of $^{12}C$, $^{16}O$, $^{22}Ne$ and $^{28}Si$ projectiles with emulsion nuclei at JINR incident energy are fairly fitted by a single Gaussian distribution. These results show that the shower particles are emitted isotropically from the decay of a single fireball as previously observed in Refs. [16, 17].
2. The statistical parameters (M, $\sigma^2$, S and K) show a gradual enhancement with increasing projectile mass number, assuming that the incident energy is held constant. This may be due to the increasing number of participating nucleons from the projectiles. The values of M ($\overline{\eta}$ ) >0, indicate that the generated particles are generally emitted in the forward direction.
3. PRDs have negative skewness values. This means that the experimental data are left skewed, while the distributions have positive kurtosis values. The value of the scaled variance $\omega(\sigma^2/M)$ slightly increase with increasing the projectile mass number.
4. The minor deviation between the experimental data and the fitting curves at the forward tails are be due to leading projectile protons contamination in the forward direction of emission Refs. [17, 18].
5. The values of $\sigma^2$ increase with increasing the projectile mass number at 3.7A GeV. The deviation in case of $^{22}Ne$-Em interactions may be due to the incident energy difference from other projectiles.

### 4.2 Target Size Dependence

In nucleus–nucleus collisions the number of shared nucleons from both target and projectile depends on the impact parameter. The impact parameter is strongly related to the centrality of the collision system. In emulsion experiments the multiplicity of the heavily ionizing emitted particles $N_h$ is selected as one of the suitable parameters to represent the centrality of the collision system. First, we divided the present data of $^{16}O$ – Em, into two major groups of events with $1 \le N_h \le 7$ and $N_h \ge 8$. The events in the first group may represent the interactions of $^{16}O$ with the CNO emulsion nuclei in addition to the peripheral interactions with AgBr nuclei, while the second group of events represents the interactions of $^{16}O$ with the AgBr nuclei. To investigate the centrality, we further subdivide the second group of events into three subgroups: $8 \le N_h \le 15$, $16 \le N_h \le 27$ and $N_h \ge 28$.

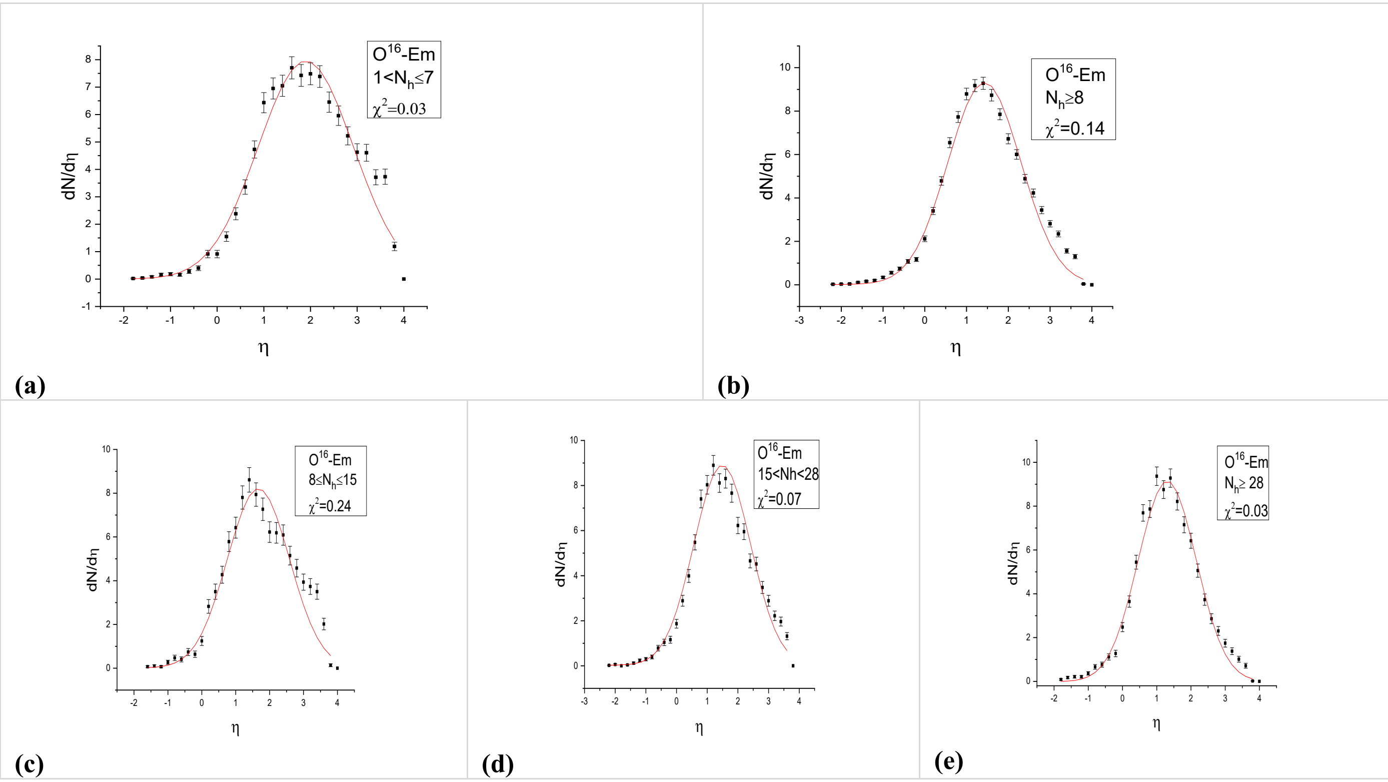


**Fig (2): (a) – (e): the experimental PRDs of charged pion produced in 3.7A GeV $^{16}O$ – Em interactions (closed square) and the corresponding Gaussian fitting (smooth curves) for different group of events (a) $1<N_h \le 7$, (b) $N_h \ge 8$ , (c) $N_h$ (8 – 15), (d) $N_h$ (16 – 27) and (e) $N_h \ge 28$. Only statistical errors are shown. All distributions are normalized to the emitted particles (100%).**

In Figures [2 (a) – 2(e)], we depict the PRDs, in 3.7A GeV $^{16}O$ interactions with emulsion nuclei according to the different above subgroups. The solid curves in the figures are the Gaussian fitting distributions, while the experimental data are the black points. The corresponding statistical calculated moments (M, $\sigma^2$, S, K and $\omega$) for each $N_h$ group of events are listed in Table (3). It can be seen that the mean values of the shower particles $<n_s>$ increase with increasing $N_h$ values [12] i.e. the values of $\rho_{max}$ increase with increasing $N_h$-values.

From Figures [2(a) – 2(e)] and Table (3), it can be noted that:

1. The value of mean pseudorapidity M (= $\overline{\eta}$ ) decreases slowly with $N_h$. This indicates that the cascading effect i.e. the created pions scattering on another target nucleon before emission becomes more pronounced as the target size increases. This conclusion is consistent with that given in Ref. [24]. The value of M (= $\overline{\eta}$ ) for the group of events having $N_h$ (0, 1) is nearly twice the value for the group of events having $N_h \geq 28$.
2. The value of the variance $\sigma^2$ decreases with increasing $N_h$. The variance -to- mean value ratio $\omega \left(= \frac{\sigma^2}{M}\right)$, which characterizes the fluctuations of the distributions, is decreases slowly with increasing $N_h$ values.
3. All experimental data of PRDs, for different groups of events are fitted by a single Gaussian distribution with peak position at $\eta \approx 1.5$.
4. It is also noticed that the values of $\rho_{max}$ increase with the centrality ($N_h$-values). Therefore, the number of participating nucleons increase with $N_h$ values and thus <ns> increases. For the different groups of events, the values of S increase with $N_h$ and have negative values. This means that the shapes of the distributions are nearly uniform. Furthermore, the values of K for all groups of events are positive and the excess kurtosis (K-3) are negative. This indicates that the distributions are leptokurtic with longer flat tails.
5. The deviation of the experimental data of the PRDs in figures [i.e. 1(a) – 1(e)] from the single Gaussian shape may be due to the fact that the projectile size ($^{16}O$ nucleus) is comparable to the target size ($N_h \approx$ CNO nuclei in the emulsion) but in case of projectile size smaller than target size (AgBr nuclei, $N_h \geq 8$) two Gaussian distributions may be used to fit the experimental data. These results are consistent with those of Refs. [17, 18].

**4.3 PRDs for different centralities**

Here, we direct our attention to the study of the PRDs of the shower particles produced in various collision geometries. We present the dependence of the average number of the heavily ionizing particles $<N_h>$ on the number of the noninteracting PFs, Q, which characterizes the volume of the projectile nucleus that is not overlapping with the target nucleus. Centrality of the collisions increases as the value of Q decreases and vice versa. Figure (3) illustrates this dependence for the present $^{16}O$ -Em interactions at 3.7A GeV. The experimental data are fitted by the exponential law of the form $<N_h> = a \exp(bQ)$ ( $\chi^2$ =0.79) where:

$$a = (28.31 \pm 1.17) \text{ and } b = (-0.27 \pm 0.01)$$

The obtained results support the nuclear limiting fragmentation hypothesis introduced by Beneck et al. [1], in which both projectile and target nuclei fragments are independent of each other, when the rapidity interval between them is more than 2 units.

The statistical moments M (= $\overline{\eta}$), $\sigma^2$, S, K in addition to the maximum pseudorapidity density $\rho_{max}$ and the scaled variance $\omega$ ( = $\sigma^2$/M) of the PRDs in different groups of events characterized by the Q-values (Q = 0, 1 , … 8) in the $^{16}O$- Em interactions at 3.7A GeV are listed in Table (4).

**Table (3): Values of the different moments M (= $\overline{\eta}$), $\sigma^2$ S, K, Excess (K-3) and scaled variances $\omega$ and the maximum pseudorapidity density $\rho_{max}$ of PRDs of emitted shower particles at different ($N_h$ – values) in the interactions of 3.7A GeV $^{16}O$ with emulsion nuclei.**

| $N_h$ | M (= $\overline{\eta}$), | $\sigma^2$ | S | K | Excess (K-3) | $\omega = \left(\frac{\sigma^2}{M}\right)$ | $\rho_{max}$ |
|---|---|---|---|---|---|---|---|
| 0, 1 | 2.27±0.07 | 4.23±0.1 | -0.68±0.07 | 0.86±0.14 | -2.14 | 1.86 | 0.72 |
| 1 – 7 | 1.98±0.02 | 2.96±0.04 | -0.35±0.03 | 0.37±0.07 | -2.63 | 1.50 | 1.66 |
| 8 – 15 | 1.81±0.03 | 2.28±0.04 | -0.40±0.04 | 1.59±0.09 | -1.41 | 1.26 | 2.52 |
| 16 – 27 | 1.61±0.02 | 1.69±0.02 | -0.07±0.02 | 0.07±0.07 | -2.93 | 1.05 | 5.26 |
| ≥ 28 | 1.42±0.02 | 1.19±0.03 | -0.11±0.03 | 0.62±0.06 | -2.48 | 0.84 | 9.01 |
| ≥ 8 | 1.57±0.01 | 1.56±0.01 | -0.15±0.02 | 0.61±0.04 | -2.39 | 0.99 | 5.24 |

The <$N_h$> value for each class of events are listed also in the last column in the given table. It is now possible to study the PRDs of the produced shower particles in nearly central collision in the group of events having Q = 0. Fig. (4) presents the PRDs for the shower particles emitted from the present of $^{16}$O-Em interactions at 3.7A GeV characterized by Q=0. To further investigate the above central events, we also select on the values of $N_h$. This means that there is complete overlap between the incident $^{16}$O nuclei and the different target nuclei. The calculated statistical moments for the PRDs in addition to the <$N_h$> values for group of events having $N_h$ [(1-7), (8-15), (16–27) and ≥ 28] with Q = 0 in $^{16}$O – Em interactions at 3.7A GeV are listed in Table (5).

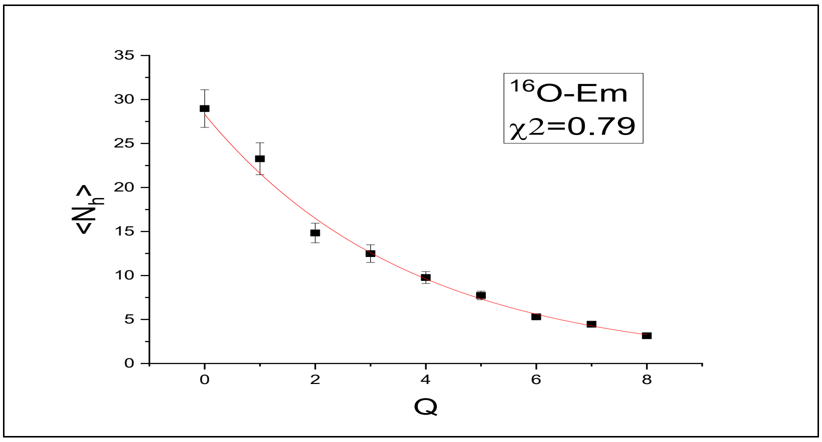


**Fig. (3): Variation of <$N_h$> as a function of noninteracting PFs, Q for $^{16}$O-Em interactions at 3.7AGeV.**

**Table (4): Values of the different moments M (= $\overline{\eta}$ ), $\sigma^2$ , S, K and <$N_h$> as well as the values of scaled variance for the PRDs $\omega = \left(\frac{\sigma^2}{M}\right)$ of the shower porticles in $^{16}$O-Em interactions at 3.7A GeV in different centrality classes characterized by Q-values (projectile spectator charge)**

| Q-value | M (= $\overline{\eta}$ ) | $\sigma^2$ | S | K | Excess (K-3) | $\omega = \left(\frac{\sigma^2}{M}\right)$ | <$N_h$> |
|---|---|---|---|---|---|---|---|
| 0 | 1.45±0.02 | 1.32±0.02 | -0.76±0.03 | 0.36±0.07 | -2.64 | 0.91 | 28.97±2.14 |
| 1 | 1.61±0.03 | 1.66±0.06 | -0.10±0.04 | 0.17±0.08 | -2.82 | 1.03 | 23.26±1.81 |
| 2 | 1.74±0.03 | 2.10±0.04 | -0.18±0.05 | 0.43±0.09 | -2.57 | 1.21 | 14.83±1.12 |
| 3 | 1.82±0.04 | 2.24±0.04 | -0.28±0.05 | 0.38±0.10 | -2.62 | 1.23 | 12.48±0.10 |
| 4 | 1.91±0.05 | 2.66±0.04 | -0.20±0.05 | -0.27±0.10 | -3.27 | 1.39 | 4.77±0.18 |
| 5 | 1.97±0.05 | 2.82±0.04 | -0.06±0.06 | 0.23±0.11 | -2.77 | 1.43 | 7.73±0.49 |
| 6 | 1. 94±0.05 | 2.76±0.05 | -0.27±0.07 | 0.13±0.11 | -2.87 | 1.42 | 5.30±0.32 |
| 7 | 1.89±0.06 | 2.46±0.06 | -0.54±0.08 | 0.13±0.10 | -2.87 | 1.30 | 4.46±0.25 |
| 8 | 1.94±0.11 | 2.72±0.08 | -0.36±0.13 | -0.30±0.20 | -3.30 | 1.40 | 3.15±0.28 |

**Table (5): Values of the different moments M (= $\overline{\eta}$), $\sigma^2$, S, K, Excess (K-3), <$N_h$> and scaled variances of the PRDs of shower particles at different groups of events ($N_h$) with Q = 0.**

| Group of events | M (= $\overline{\eta}$) | $\sigma^2$ | S | K | Excess (K-3) | $\omega = \left(\frac{\sigma^2}{M}\right)$ | <$N_h$> |
|---|---|---|---|---|---|---|---|
| $N_h$ (1-7) & Q = O | 2.01±0.09 | 3.24±0.07 | -0.02±0.01 | -0.51±0.21 | -3.51 | 1.61 | 4.50±0.83 |
| $N_h$ (8-15) & Q = O | 1.77±0.07 | 2.34±0.06 | -0.09±0.01 | -0.40±0.20 | -3.40 | 1.32 | 12.01±0.88 |
| $N_h$ (16-27) & Q = O | 1.57±0.03 | 1.61±0.3 | -0.07±0.05 | 0.33±0.10 | -2.92 | 1.03 | 23.61±3.15 |
| $N_h$ ≥ 28 & Q = O | 1.38±0.02 | 1.14±0.02 | -0.10±0.03 | 0.85±0.07 | -2.15 | 0.83 | 35.82±3.51 |

As a result, we plot the experimental PRDs of the shower particles for the above groups of events in Figures 5 (a) - 5 (d). The smooth curves in the figures are the Gaussian distributions fitting the experimental data (closed squared) for each group of events.

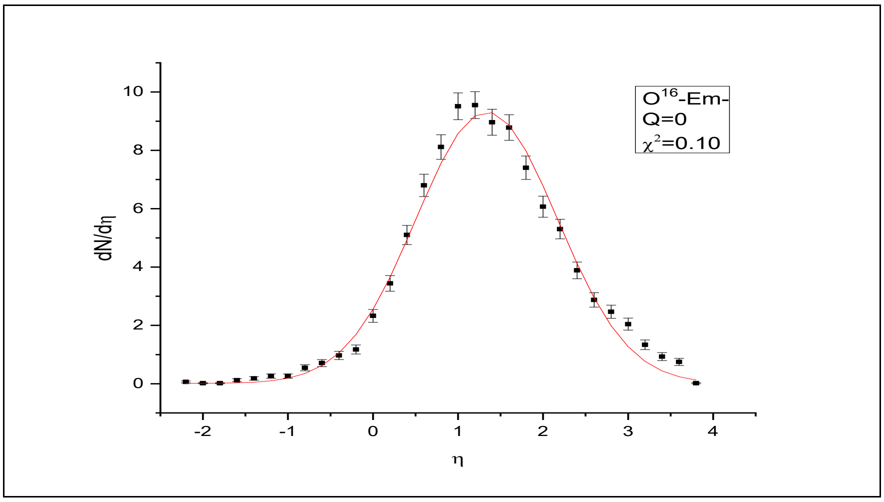


**Fig. (4): The PRD of the pion production in $^{16}$O-Em collisions at 3.7AGeV and the corresponding Gaussian distribution for group of events having Q=0. All distributions are normalized to the emitted particles (100%).**

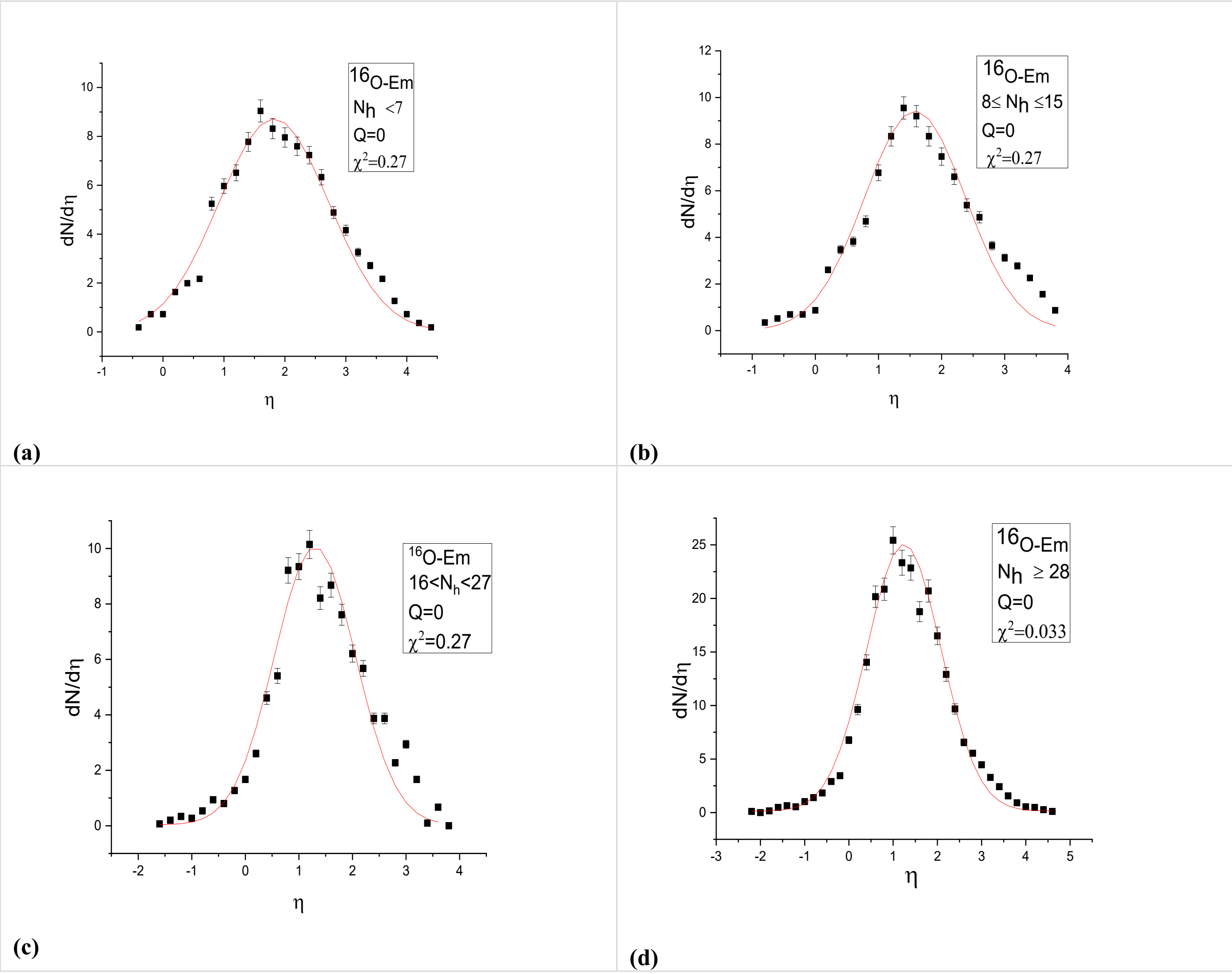


**Fig. (5): PRDs of the emitted charged pion produced in 3.7A $^{16}$O-Em collisions and the corresponding Gaussian distribution (solid curve) for different group of events: (a) $N_h \leq 7$, (b) $8 \leq N_h \leq 15$, (c) $16 \leq N_h \leq 27$ and (d) $N_h \geq 28$ with Q = 0. Only statistical errors are shown. All distributions are normalized to the emitted particles (100%).**

From Fig. (4), and Fig. (5), the calculated moments in Table (4) and table (5), one can notice that:

1. The mean pseudorapidity M increases with increasing the number of noninteracting PFs, Q, i.e., the values of M decreases with increasing the degree of centrality of the collisions. Also, the values of M are greater than 0, since we are working in the target frame of reference (the lab. system). These results reflect the cascading effect, which increases with decreasing impact parameter. This result means that, the peak of the distribution shifts towards the right as the centrality decreases.
2. The values of the variance $\sigma^2$ of all PRDs groups increase with increasing the Q-values, i.e., increase with increasing the mean value M.
3. The experimental PRD of the shower particles produced for each group of events are fitted by a single Gaussian shape with small skewness S (-ve value). This result reflects that all distributions are symmetric in the shape about the central value (M) with small tail extending in the negative $\eta$ range. Therefore, the values of the calculated kurtosis K are generally nearly independent of the degree of centrality of the collision. These results for S and K support single Gaussian distribution for the various PRDs at Dubna energy.
4. The scaled variance $\omega$ $\left(=\frac{\sigma^2}{M}\right)$ increases with increasing the impact parameter (Q-value). These results show that the PRDs for the different groups of the event are narrower in shape with clear peaks. The peaking position shifts to lower values with decreasing the impact parameter.
5. The values of the $< N_h >$ for the different groups of event increase with increasing degree of centrality. The maximum value of $<N_h>$ corresponds to group of events having Q = 0 (central collisions). This notion has been previously supported by Refs. [26, 27].

## 5- SUMMARY AND CONCLUSIONS

The shower particles PRDs in 3.7A GeV $^{16}O$ interactions with emulsion nuclei are analyzed according to the degrees of centrality, depending on the values of the number of target disintegration particles $N_h$ and the values of total charge Q of the noninteracting PFs in each event, i.e., the relation between the collision geometry parameter and the centrality degree is investigated.

Our data are classified into different groups. The PRDs of shower particles in each group of events are fitted by a single Gaussian distribution. For each specified group of events we calculate the statistical moments including the mean pseudorapidity value M ($=\overline{\eta}$ ), variances $\sigma^2$, skewness (S) and Kurtosis (K). The present data from interactions of $^{16}O$ projectile with emulsion nuclei are compared with the corresponding data from the interactions of other projectiles $^{12}C$ and $^{28}Si$ at 3.7A GeV as well as $^{22}Ne$ interactions at 3.3A GeV. [17-18]

From the above study the following conclusions are drawn:

1. The PRDs of the shower particles emitted from interactions of different projectiles ($^{12}C$, $^{16}O$, $^{22}Ne$ and $^{28}Si$) with emulsion nuclei can be fitted with single Gaussian distributions in the pseudorapidity range $\eta > 0$.
2. The values of the calculated statistical parameters M ($= \overline{\eta}$), $\sigma^2$, S and K increase with increasing the projectile mass number $A_{proj}$ at nearly the same incident energy. The increase in the value of M with increasing $A_{proj}$ may be due to the increase of contamination of the leading protons in the forward direction of the events.
3. For all PRDs. the values of scaled variance $\omega(= \sigma^2/M)$ are slightly increasing with increasing of $A_{proj}$.
4. The values of the skewness are small and negative, meaning that the PRDs of the created pions have symmetric shapes with small tail extending to the negative region ($\eta < 0$) (i.e out of the kinematic possible range). The pions emitted beyond the kinematics in the backward directions with $\eta < 0$ may be emitted by another mechanism (decaying source) which is completely different from the main source of the pion creation (hadronic matter) in high energy nucleus – nucleus collisions [25, 26].
5. The values of the excess kurtosis (K-3) for all PRDs are negative, indicating that all distributions are platykurtic in shape with shorter tails.
6. The present $^{16}O$-Em interaction events are further classified into groups with various degrees of centrality depending on the values of both $N_h$ and/or Q. The PRDs of the created pions in each group have been fitted by a Gaussian distribution in the range $0 \leq \eta < \eta_p$.

7. In $^{16}O$-Em interaction data, the values of the mean pseudorapidity M (= $\bar{\eta}$) decrease slowly with increasing $N_h$ – values. This reflects the growing of the cascade effect with increasing target size. The value of M for the group of events having $N_h$ (0,1) which corresponds to pure peripheral collisions is nearly twice the corresponding value for the group of events having $N_h \geq 28$ (more central collisions). On the other hand, the values of M are found to increase with decreasing the centrality degrees (i.e., with increasing total charge of emitted noninteracting PFs).

8. The variance of the Gaussian distribution $\sigma^2$ decreases with increasing $N_h$–values and vice versa. The values of $\sigma^2$ also increase with increasing Q-values (peripheral collisions). In other words, the values $\sigma^2$ decrease as the centrality increases.

9. The values of the scale variance ω (= $\sigma^2$/M) decrease with increasing the $N_h$– values, i.e., the values of ω decrease with increasing the degree of centrality.

10. From the investigation of the calculated statistic parameters S, K and the excess kurtosis (K-3) for the PRDs shapes of the pion production in different centrality groups of $N_h$ or Q, we found that the values of the skewness (-ve values) increases with increasing the centrality degree, reflecting that the distributions have platykurtic shape with tails in the negative pseudorapidity range. In conclusion the PRDs of the shower particles emitted from nucleus-nucleus collisions at Dubna energy can be describe by single Gaussian shape.

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