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Investigation of Thermal Hydraulics Parameters of VVER-1000/V-320 during LOCA Accident

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ABSTRACT

This paper presents a model of the VVER-1000 reactor using data from KNPP/ VVER -1000/ V-320, the VVER-1000 coolant transient benchmark, and previous studies. The RELAP5/MOD3.2 code, commonly used in thermal hydraulics simulations, was utilized. Model validation included a steady-state analysis, with results closely aligning with the plant specified parameters. Following validation, LOCA scenario was simulated to evaluate its impact on thermal hydraulic parameters and the reactor's behavior during severe accident conditions. In the LOCA simulation, the core region was identified as the most critical area for detecting thermal behavior during severe accident conditions. The cladding temperature distribution indicated a rise in temperature within the first seconds of the accident. Comparing temperatures between normal operation and LOCA conditions highlighted the behavior of the primary coolant during an accident. The pressure distribution across the core region revealed a pressure drop in the primary circuit and core dewatering. Overall, the RELAP5 model results demonstrated a good approximation to the plant behavior during an accident.

NOMENCLATURE

KNPP	: The Bulgarian Kozloduy Nuclear Power Plant
LOCA	: Loss of Coolant Accident
VVER	: The Water-Water Energetic Reactor
PWR	: Pressurized Water Reactor
LOFA	: Loss of Flow Accident
SFP	: Spent Fuel Pool
DNB	: Departure from Nucleate Boiling
TMA	: Thermal Mechanical Analysis
RPV	: Reactor Pressure Vessel
RV	: Reactor Vessel
NPP	: Nuclear Power Plant
PHTS	: Primary Heat Transport System
BARs	: Burnable Absorber Rods
DBAs	: Design Basis Accidents
WCRs	: Water Cooled Reactors

1. INTRODUCTION

Thermal hydraulics is a vital field that examines how fluids behave under the influence of heat, particularly in complex systems where fluids may consist of multiple components or exist in different phases, such as liquid and gas. This field plays an essential role in various engineering applications, including power plants, nuclear reactors, and industrial processes. Thermal hydraulics analysis is especially important for maintaining the safety and efficiency of nuclear reactors by managing heat and ensuring proper cooling to prevent overheating and potential accidents [1].

The VVER is a series of PWR first developed in the Soviet Union (now Russia) before the 1970s. These reactors have undergone continuous upgrades over the years.

The VVER-series represent a major step forward in nuclear reactor technology, offering both significant power generation capacity and a range of safety advancements. It exemplifies the progress made in reactor development by combining high output with enhanced safety features. Its widespread use and long operational history highlight its importance in the global

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nuclear energy sector. Continuous upgrades and safety innovations have made the VVER-series standard in modern nuclear reactors design.

For example, Seyed Khalil Mousavian et al.[2] conducted a simulation of LOFA for the VVER-1000/V446 SFP using RELAP5 and MELCOR 1.8.6 for both best estimate and severe accident analysis. The spent fuels decay heat was calculated using the ORIGEN-II code at varying power levels. The simulation examined key stages such as pool water heating, boiling, water level reduction, uncovering of the spent fuels, temperature increases, oxidation onset, fuel melting, and radionuclide release. Their findings showed reasonable consistency between the RELAP5 and MELCOR results, particularly up to the start of oxidation. Similarly, M. Bottcher et al.[3] utilized four different CFD codes (ANSYS Fluent, ANSYS CFX, Trio CFD, and STAR-CCM) to simulate the mixing of the coolant in the VVER-1000/V320 reactor vessel at KNPP. They applied two distinct methods for modeling the upper plenum and a simplified model for the RPV. These simulations were part of the VVER-1000 coolant transient benchmark (V1000CT-2) mixing exercise. The results from the different codes and models were compared to evaluate their consistency and accuracy with experimental data. Overall, the findings showed strong agreement across the codes and models, with only minor discrepancies. The simplified models effectively captured the overall coolant mixing behavior and provided further insights into local flow structures and mixing dynamics.

While Marek Ruscak et al.[4] used the MELCOR 1.8.6 code to simulate accident scenarios that could lead to core melting, including a station blackout (SBO), which results in a total loss of alternating current power at a NPP. They also analyzed four other scenarios that combined SBO with additional technological failures, such as the loss of the steam generator feedwater system and small, medium, and large break LOCAs. Their study provided detailed insights into the progression of these accidents, examining temperature and pressure changes, hydrogen generation, and the release of molten corium and debris into the plant containment. And, Jan Syblik et al.[5] conducted simulations of both steady state and transient conditions, specifically focusing on a LOFA in the VVER-1000 reactor. They emphasized the use of subchannel codes as advanced tools for determining safety margins and key thermal hydraulic parameters like the departure from nucleate boiling ratio. For their analysis, they used SubChanFlow 3.5 and VIPRE-01 to simulate fuel assembly and compared the results as a benchmark. SubChanFlow was generally found to be more conservative than VIPRE, making it suitable for future transient analysis evaluations. Differences in the results were attributed to variations in the crossflow models used in the subchannel codes. Despite these variations, the DNB ratio, calculated with the OKB correlation, remained within safety limits. However, Ahmed M. Refaey et al.[6] focused on the TMA of the lower plenum of the RPV in the VVER-1000 during the later stages of a severe accident. They stressed that ensuring the structural integrity of the RPV during accidents is key to extending the operational life of NPP. Using ANSYS FLUENT 17.2, they analyzed radial and axial temperatures and stress distribution across the vessel wall. Boundary conditions from the ASTEC code were used to model corium behavior in the lower plenum. The study examined how stress, strain, and damage evolved at critical axial layers, identifying potential crack locations. Their findings suggested that without external vessel flooding, the risk of thermal failure (melt-through) is high due to the heat from decay power. However, external flooding could improve vessel stability, potentially preventing failure and allowing retention of molten corium within the RPV.

Numerous studies over the years have concentrated on simulating and modeling the VVER-series reactors using a variety of tools and techniques. These advanced modeling tools help ensure that VVER-series reactors run safely, efficiently, and in line with regulatory requirements. The insights gained from these simulations are crucial for the ongoing enhancement of reactor technology and safety protocols.

This paper aims to develop detailed thermal hydraulics models for NPP safety analysis. It reviews and discusses the results of these models by comparing the VVER-1000/V-320 reactor against benchmark results to verify accuracy. A model of a specific nuclear accident scenario is then created using these validated models to illustrate the accident's effects on thermal hydraulic parameters.

The model was developed using data from the KNPP VVER-1000/V-320, the VVER-1000 coolant transient benchmark, and relevant scientific literature based on earlier benchmarks [7]. The thermal hydraulics code RELAP5, a widely used tool in the field, was utilized for this analysis.

RELAP5 is a sophisticated simulation tool that enables the modeling of the coupled dynamics of the reactor coolant system and the reactor core during various operational transients and hypothetical accident scenarios in nuclear reactors. It is extensively used for reactor safety analysis, reactor design, operator training through simulators, and as an educational resource in academic institutions. Originally developed by the Idaho National Engineering Laboratory (INEL) for thermal hydraulics studies in Pressurized Water Reactors (PWRs), RELAP5 is designed to provide best-estimate transient simulations of light water reactor coolant systems during postulated accidents. Furthermore, it serves as the basis for a nuclear plant analyzer (NPA). The RELAP5 model employs a one-dimensional, two-fluid approach to two-phase flow, capable of simulating non-equilibrium, non-homogeneous conditions, and includes a model for the transport of non-condensable gases through the system [8].

2. REACTOR DESCRIPTION

The VVER-1000 is a distinguished NPP design, renowned for its power output and safety features. It is extensively used across the globe, with 31 units in operation and a total of around 500 reactor-years. The VVER-1000/V-320, the standard design from the early 1980s, is in service at eight different locations. This reactor model has an outstanding safety record, with no major safety incidents reported.

The RPV serves as the containment boundary for both the reactor core and the high-pressure coolant. Its detailed structure is illustrated in Figure 1. The lower section of the core barrel contains an elliptical flow distributor plate with perforations that extend through the inner vessel. These perforations feature 1,344 circular holes, each measuring 40 mm in diameter [9].

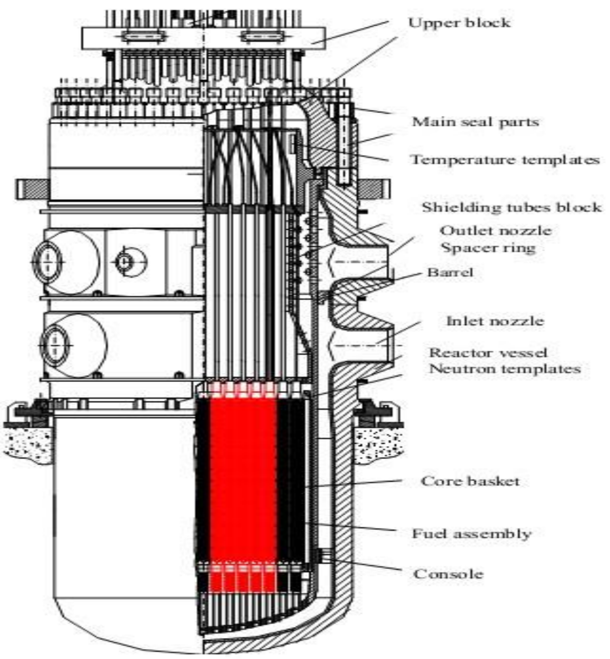

Fig (1): Reactor Vessel [9]

This design maintains a large flow area even during extreme LOCA and high temperatures within the vessel. The expanded flow area ensures that a significant portion of the flow path remains unobstructed, enabling coolant to reach the core's active region under severe conditions. After the coolant flows through the perforations in the inner vessel, it moves into the lower plenum, which is located next to the fuel support columns. These columns are also perforated to allow coolant to enter the fuel assemblies.

The PHTS carries heat from the reactor through four parallel loops to horizontal steam generators. Figure 2 shows the layout and elevations of this primary circuit. In the VVER-1000/V-320, the hot leg nozzle for each loop is positioned directly above the cold leg nozzle for the corresponding primary loop on the RPV [9].

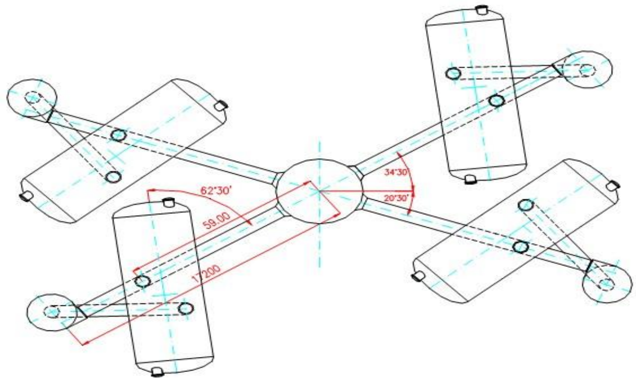

Fig. (2): Primary circuit loops – layout [9]

3. METHODS AND SCENARIOS

3.1. VVER-1000 RELAP5 model

An integrated plant model was created to address the VVER-1000 Main Coolant Pump (MCP) shutdown scenario, covering both steady-state and transient conditions using the RELAP5 thermal hydraulics tool. Figure 3 shows the detailed nodalization scheme for the RELAP5 model. The technical data for this model were obtained from the KNPP VVER-1000/V-320 and the VVER-1000 Coolant Transient Benchmark.

The reactor core consists of 163 fuel assemblies arranged in a hexagonal pattern with a 23.6 cm pitch. Each fuel assembly contains 311 fuel rods, 18 central guiding channels for control rods and/or BARs, and a central channel. These elements are arranged in a triangular pattern with a 1.275 cm pitch. The fuel rods are composed of UO2 pellets. The specifications of the core used in this study are provided in Table 1.

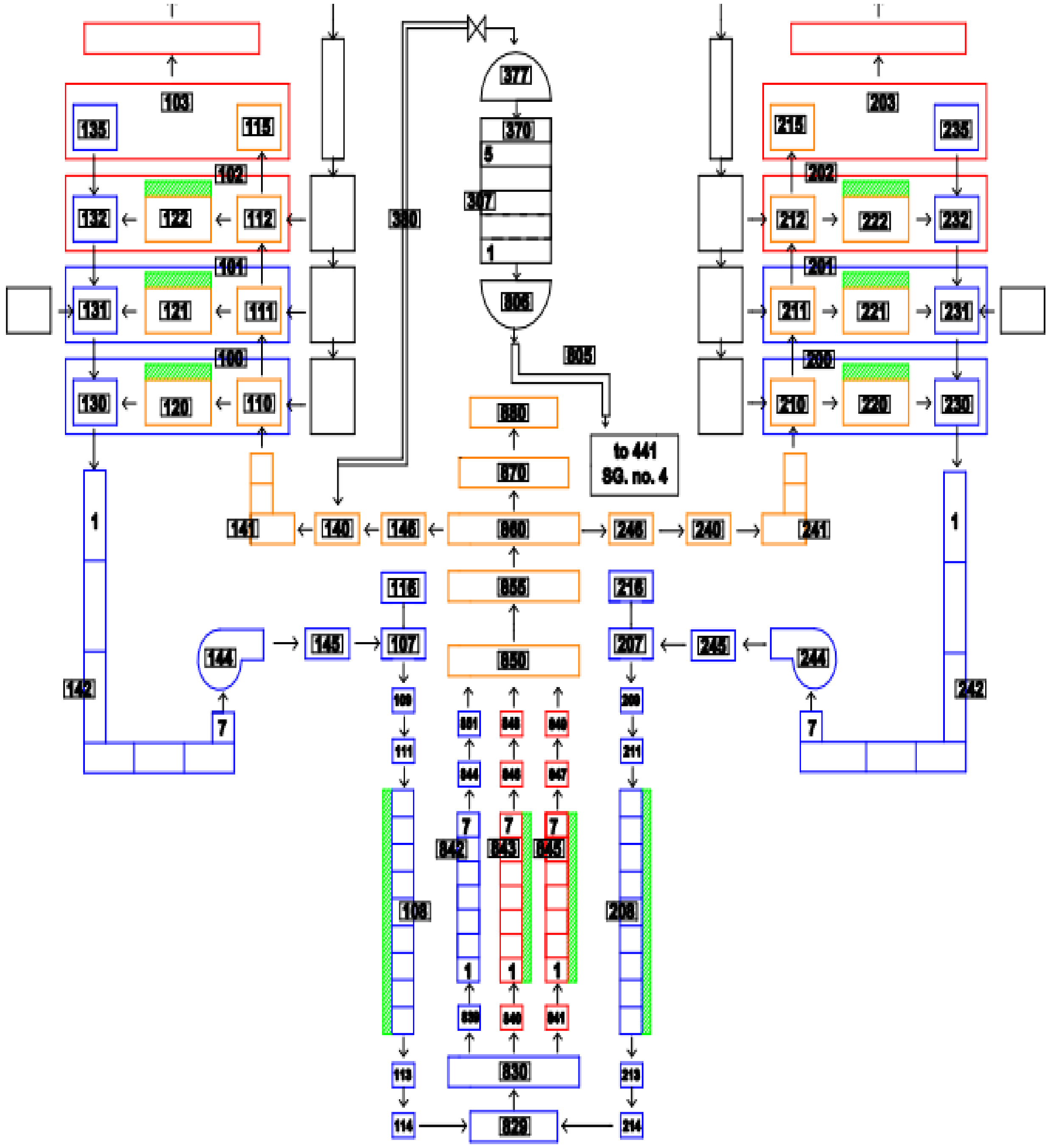

Fig. (3): VVER-1000 RELAP5 four loops model

The core was modeled using three parallel volumes: the core hot channel, the average channel, and the core bypass. Heat produced in the fuel pins is transferred to the primary coolant through heat structures linked to both the hot and average channels. The primary circuit is divided into five key systems: *the Hot Leg Piping System*, which transports heated coolant from the reactor core to the steam generators; *the Cold Leg Piping System*, which returns cooled coolant from the steam generators back to the reactor core; *the pumps*, which circulate coolant throughout the primary circuit; *the steam generators system*, which transfers heat from the primary coolant to the secondary side to produce steam for the turbine; and *the pressurizer*, which maintains pressure in the primary circuit to prevent coolant from boiling.

This integrated configuration ensures effective heat transfer and circulation in the VVER-1000 reactor, enabling safe and stable operation across different conditions.

Table (1): VVER-1000 Reactor specifications [10]

Item	Data
Total core power output (MWt)	3000
Core pressure (MPa)	15.7
Core pressure drop (MPa)	0.142
Coolant operating temperature - inlet (°K)	564.15
Coolant operating temperature - outlet (°K)	592.05
Reactor coolant flow (kg/s)	17 611
Number of fuel assembly (FA)	163
Number of fuel rods in FA	311
Fuel rod pitch (mm)	12.75
Fuel pellet outside diameter (mm)	7.57
Cladding inner diameter (mm)_	7.73
Cladding outer diameter (mm)	9.1
Height of active fuel region (mm)	3 550
Fuel pellet material	UO_2
Cladding material	alloy Zr + 1% Nb

3.1. Model Validation

To verify and approve the node schemes of a NPP, a complex procedure known as the validation process is employed. This process aims to confirm that nuclear codes operate correctly in comparison with experimental results and to assess the reliability of the code outputs. As NPP designs advance and the demand for safer plants grows, the variety and complexity of nuclear codes have expanded, making the validation process more intricate [11]. A nodal scheme for a plant is regarded as validated when it accurately represents the geometry of the modeled system, accurately reflects the nominal measured steady-state conditions, and functions reliably in time-dependent scenarios. *Steady-State Validation (Initial Qualification)* involves verifying that the structural and operational data meet the nominal measured steady-state conditions. This phase ensures that the model faithfully represents the system's steady-state behavior [7].

Steady-State Validation

Steady-State Validation seeks to ensure that the model’s structural and operational data precisely represent the system's performance in a steady state. This phase is crucial for establishing the model's baseline accuracy before it is tested under dynamic or transient conditions. A successful validation indicates that the model can accurately simulate the system’s behavior under stable and transient operating conditions. Table 2 provides a comparison between the steady-state results and the design values for the Kozloduy VVER-1000 plant.

Table (2): Nominal Full-Power Steady-State Primary System Parameters and Calculated Values

Parameter	KNPP value	RELAP5 results	Deviation %
Core Power, MW (th)	3000	3000	0.0
Primary pressure (top volume), MPa	15.65	15.7049	0.3508
Pressurizer Temperature (K)	620.0	618.536	-0.2361
Coolant temperature at reactor inlet (K)	560.0	562.486	-0.4439
Coolant temperature at reactor outlet (K)	593.0	593.648	0.1093
Nominal coolant flow (kg/s)	17610	17385.32	-1.2759
Primary pressure at SG inlet, (MPa)	15.64	15.6452	0.03325
Coolant temperature at SG inlet (K)	591.0	588.803	-0.3717
Coolant temperature at SG outlet (K)	560.0	563.021	0.539

Table 2 demonstrates that the calculated results fall within the design uncertainty range for all specified parameters. This comparison confirms that the model accurately represents the plant's actual steady-state operating conditions. By matching the model's results with the design specifications, the table underscores the model's effectiveness in replicating the Kozloduy VVER-1000 plant's steady-state behavior and verifies that it aligns with the anticipated design values.

3.2. Accident Scenario

One of the DBAs for WCRs is the LOCA due to the failure of a large coolant pipe. Specifically, for a PWR, the DBA is typically initiated by the double-ended guillotine break of a large coolant pipe between the RV and the main circulation pump. In the case of the VVER-1000 reactor, the scenario considered is a both-sides break of the main coolant pipeline (850 mm in diameter) at the reactor inlet, which serves as the starting event for a LOCA. The breakage of a large-diameter primary pipeline causes a massive blowout of coolant, leading to a rapid drop in primary circuit pressure and dewatering of the core. Consequently, heat removal from the core deteriorates, resulting in a sharp increase in fuel cladding temperature, with a rate of approximately 102 K/s. An emergency protection signal is triggered almost immediately after the primary circuit parameters reach the corresponding thresholds, occurring just 0.1 seconds after the accident starts. This results in a decrease in reactor power down to the residual heat level, due to the reduction in coolant density in the core and the response of the emergency protection system.

To study the behavior of nuclear reactors under such accident conditions, a detailed RELAP5 model was developed to simulate the LOCA scenario at the KNPP (VVER/1000 – V-320). The model is based on the previously validated model but incorporates changes specific to the LOCA event, summarized as follows:

1. The reactor is assumed to have been operating for an extended period (approximately 2000 seconds).
2. No transient trips occur before the accident.
3. A double ended rupture of the cold leg in loop 2 (between the coolant pump and reactor vessel inlet nozzles) is assumed, Figure 4.
4. The reactor scram system is initiated to contribute to the reactor shutdown, Figure 5.

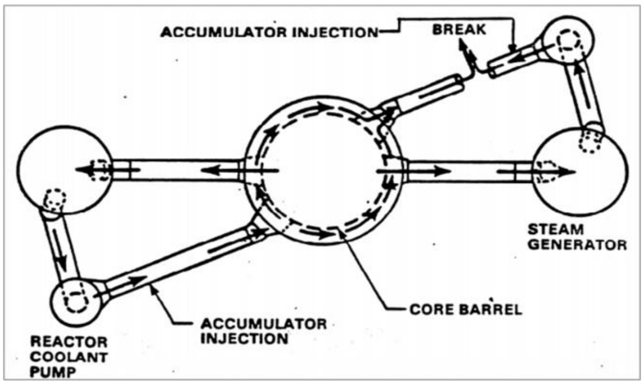

Fig. (4): Cold Leg Break Steam Flow Path [13]

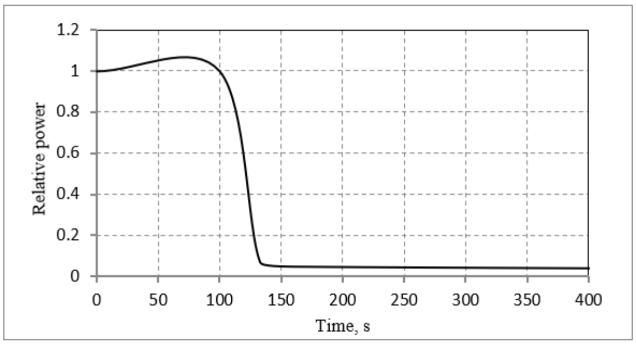

Fig. (5): Relative core power (scram initiated)

4. RESULTS AND DISCUSSIONS

Based on previous assumptions and technical data for the LOCA accident, a RELAP5 model was developed to represent the performance of the VVER-1000/V-320 during such an event. Figure 6 shows the clad temperature.

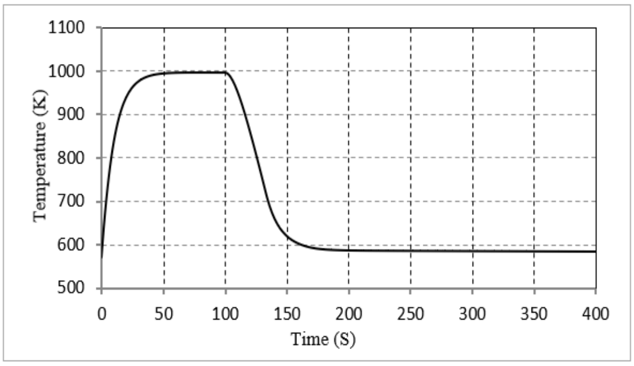

Fig. (6): Cladding temperature

As observed in Figure 6, the cladding temperature distribution can be divided into two main stages: **The first stage**, known as *the Blow-down period*, occurs within the first 100 seconds. During this phase, the cladding temperature rises sharply due to the accident. This sharp increase is a result of the primary coolant being discharged, which leads to a significant decrease in the heat removal rate from the core. Consequently, the cladding heats up rapidly. This heating can cause the cladding to swell, balloon, or even rupture. However, it is important to note that the swelling-to-rupture scenario was not modeled in this study due to uncertainties in the actual fuel behavior under these conditions. **The second stage** is *the Refill period*, during which water is injected by the low-pressure injection system. Following the blow-down period, the reactor scram system is activated, leading to a decrease in the cladding temperature. As the low-pressure injection system introduces water, the cladding temperature continues to decrease, eventually returning to normal starting conditions. The analysis of the cladding temperature distribution highlights the critical phases during an accident scenario and underscores the importance of effective cooling mechanisms to prevent severe damage to the reactor core.

A comparison of the core outlet temperature distribution under normal operation versus LOCA conditions is illustrated in Figure 7.

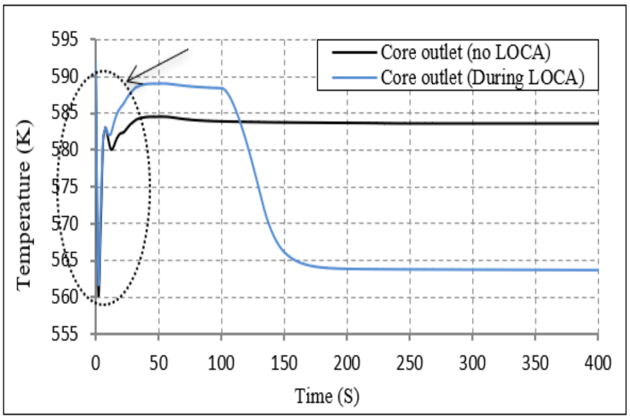

Fig. (7): Core outlet temperature during normal and LOCA conditions.

As shown in the Figure 7, during the initial period, the core outlet temperature curves follow a similar trend as the primary coolant operates under normal power conditions. During, this phase, the core outlet temperature stabilizes at a constant value as the reactor operates under steady-state conditions. The primary coolant effectively removes heat from the core, maintaining a stable temperature. However, after the accident occurs, the trends of these temperature curves diverge significantly. Under normal operation, the temperature eventually stabilizes at a constant value. In contrast, during the accident, the temperature increases by about 10 degrees compared to the normal condition. Following the activation of the reactor scram system, the temperature decreases sharply after approximately 100 seconds. This rapid decrease is due to the immediate cessation of nuclear fission reactions and the subsequent reduction in heat generation within the core. The primary coolant continues to circulate, but with the reactor shut down, the heat removal rate exceeds the heat generation rate, leading to a drop in temperature. The temperature then remains relatively constant as the reactor shuts down completely. If the scram system does not activate, the temperature will continue to increase without stopping, causing significant damage. Therefore, the scram system is crucial for ensuring the reactor's safety during an accident. A failure in the scram system could lead to severe damage, highlighting its importance in maintaining reactor safety under accident conditions.

Figure 8 similarly illustrates the core inlet temperature distribution under both normal operation and accident conditions, showing the same trend at the initial period as observed for the core outlet temperature.

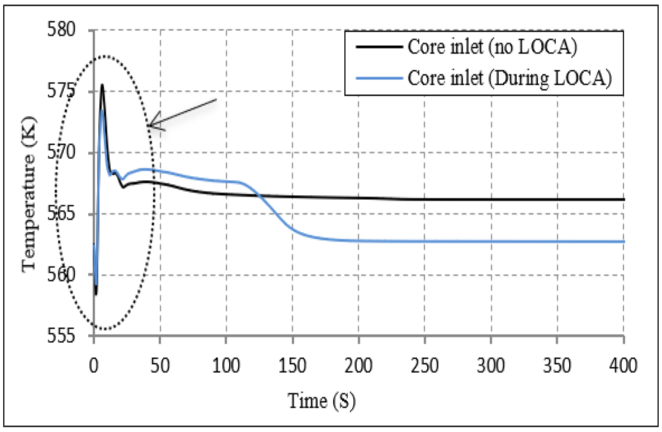

Fig. (8): Core inlet temperature during normal and LOCA conditions.

When a break occurs in the large-diameter primary pipeline, the primary coolant is rapidly lost, leading to a significant pressure drop within the primary circuit. This pressure drop is critical as it directly affects the core's ability to remain adequately cooled. As the coolant is lost, the core begins to dewater, which can severely impact the reactor's safety and stability.

Figure 9 demonstrates the core pressure distribution during this event. It shows that the pressure at the core inlet is higher than at the core outlet. This pressure gradient results in a negative pressure difference across the core, which is a crucial factor in understanding the coolant flow dynamics during the accident.

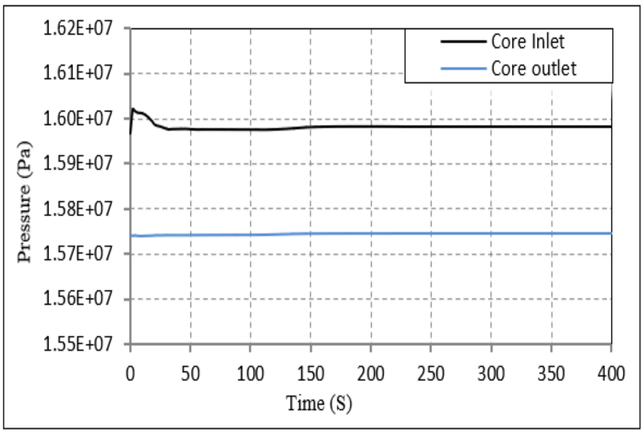

Fig. (9): Core pressure.

Figure 10 further illustrates this negative pressure difference. The higher pressure at the core inlet compared to the core outlet indicates that the coolant flow is disrupted, leading to inadequate cooling of the reactor core. This negative pressure difference is a key indicator of the severity of the coolant loss and the potential for core damage if not promptly addressed.

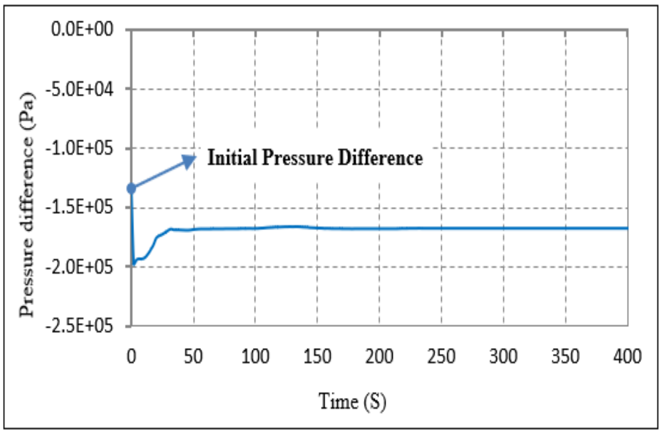

Fig. (10): Pressure difference.

CONCLUSION

The objective of this paper was to develop comprehensive thermal hydraulics models for NPP safety analysis. This was accomplished by presenting and discussing the results of the proposed models, which were used to analyze an operational VVER-1000/V-320 plant and compare them with benchmark results as part of the validation process. Following this, a model was created and validated to simulate a specific nuclear accident scenario and assess its impact on thermal hydraulic parameters.

The initial stage is a model validation, where a stable steady state was achieved, with calculated quantities closely aligning with plant-specified parameters. The results demonstrate that RELAP5/MOD3.2 is capable of predicting the thermal hydraulic behavior of the VVER-1000 reactor for the particular transients analyzed. To further investigate nuclear reactor behavior under accident conditions, a detailed RELAP5 model was developed to simulate a LOCA accident at KNPP (VVER/1000 – V-320), based on the validated model, with modifications to the assumptions related to the LOCA event.

The LOCA accident was modeled to understand how the VVER-1000 reactor behaves under accident conditions. This type of accident involves a significant loss of coolant, which is critical for maintaining the reactor's temperature and pressure within safe limits. The focus of the modeling was on key thermal properties, which are crucial for assessing the reactor's safety and performance during an accident. Monitoring the temperature within the reactor, especially in the core region, is vital. During a LOCA, the temperature can rise rapidly if the coolant is lost, potentially leading to overheating and damage to the reactor components. Therefore, the activation of the reactor scram system is essential to control the rising core temperature. Additionally, the pressure within the primary circuit of the reactor is another critical parameter. A LOCA causes a drop in pressure, which can affect the coolant flow and the reactor's ability to remove heat effectively. Thus, studying and monitoring the pressure difference is important to ensure the reactor remains safe and operational during such events. However, some properties, such as cladding hydrogen generation, debris temperature, and creep rupture, were not detected due to the complexity of cladding oxidation kinetics, non-stationary temperature regimes, cladding deformation, and loss of tightness. Additionally, water level properties were not modeled due to the challenges of using RELAP5 to assess water level conditions and the boiling state.

REFERENCES

[1] Nikolay I. Kolev, MULTIPHASE FLOW DYNAMICS 4, Nuclear Thermal Hydraulics, Handbook, Springer, 2nd edition (2012).

[2] Seyed Khalil Mousavian, Amir Saeed Shirani, Francesco D'Auria, Analysis of loss of cooling accident in VVER-1000/V446 spent fuel pool using RELAP5 and MELCOR codes., Nuclear Engineering and Technology 55 (2023) 3102 – 3113.

[3] M. Bottcher, O. Bernard, A. Mas, V. Sanchez, R. Nop, F. Belaunde, C. Bourcier, D. Ruban, A. Hashymov, O. Halim, A. Pucciarelli, N. Forgione, CFD analysis of coolant mixing in VVER-1000/V320 reactor pressure vessel, Annals of Nuclear Energy 197 (2024) 110274.

[4] Marek Ruscak, Guido Mazzini, Milos Kyncl, Sevastyan Savanyuk, Miroslav Hrehor, Alis Musa, Alain Flores y Flores, VVER 1000 Severe Accident Analyses Using MELCOR Code, Journal of Nuclear Engineering and Radiation Science 5 (2019) 030913.

[5] Jan Syblik, Alis Ruscak Musa, Milan Malacka, Guido Mazzini, SubChanFlow and VIPRE codes benchmark for VVER-1000, Nuclear Engineering and Design 418 (2024) 112936.

[6] Ahemd M. Refaey, Salwa H. Abdel-Latif, Samaa A. Wasfy, Thermal hydraulic and mechanical behaviour of VVER-1000 reactor lower plenum in the late phase of severe accident, Progress in Nuclear Energy 139 (2021) 103833.

[7] Pavlin Groudev, Malinka Pavlova, simulation of loss-of-flow transient in a VVER-1000 nuclear power plant with RELAP5/MOD3.2, Progress in Nuclear Energy 45 (2004) 1-10.

[8] SCDAP/RELAP5/MOD 3.2 Code Manual, NUREG/CR-6150, Vol. 3 (1998).

[9] Boyan Ivanov, Kostadin Ivanov, VVER-1000 Coolant Transient Benchmark, phase 1 (V1000CT-1) Vol. I: Main Coolant Pump (MCP) switching on, NEA/NSC/DOC (2002) 6.

[10] M. Amin Mozafari , F. Faghihi, Design of annular fuels for a typical VVER-1000 core: Neutronic investigation, pitch optimization and MDNBR calculation, Annals of Nuclear Energy 60 (2013) 226-234.

[11] Khalil Moshkbar-Bakhshayesh , Soroush Mohtashami, Validation of codes for modeling and simulation of nuclear power plants: A review, Nuclear Engineering and Design 421 (2024) 113-120.

[12] Nuclear Safety Report, Nuclear Fuel Behavior in Loss-of-coolant Accident (LOCA) Conditions, Nuclear energy agency, OECD (2009), NEA No. 6846.

[13] Neil Todreas, Loss of Coolant Accidents (LOCA) in PWRs: Deterministic View, Course 22.39, December (2005), Lecture 9.