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Adding a Correction Terms for the Shell of the Liquid Drop Model and the Quark-Like Model

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ABSTRACT

This paper aims to add a corrective term to Liquid Drop Model (LDM) and Quark – Like Model (QLM), which is the term for closed shells of magic nuclei. This was done by relying on valence nucleons. Fitting to the terms volume, surface, Coulomb repulsion, asymmetry, pairing and shell term in LDM was performed using the least-square method LSM by designing a code in Fortran 95 for 261 magic nuclei within the range ($2 \le Z \le 92$), this correct and balance nuclear binding energy values, especially for magic nuclei, with experimental values. A fit of the QLM has also been made in order to correct the values of the nuclear binding energy. The standard deviation (σ) was used as a statistical tool to determine the extent to which the models can be adopted to explain the behavior of magic nuclei, in addition to the high accuracy in determining the experimental nuclear binding energy. The values of the standard deviation are σ= 0.144 and σ= 0.84 for the updated formulas of the Generalized Liquid Drop Mode (GLDM) and the Generalized Quark Like Model (GQLM) respectively.

1. INTRODUCTION

The history of nuclear physics can be divided into three stages. It began with the discovery of the radioactivity of the nucleus and ended in 1933 with the discovery of nuclear fission. During this period the basic components (protons and neutrons) of the nucleus were discovered, as well as quantum laws governing its behavior [1] . The second period, from 1947 to 1969, involved the development of nuclear spectroscopy and nuclear models, and finally, the emergence of the theory of microscopic uniformity that began in the Sixties, which helped to understand the structure and behavior of protons and neutrons in terms of the fundamental interactions of their constituent particles, quarks and gluons, and this period also saw the identification of non-classical micro-mechanisms in the nuclear structure [2].

Among the essential nuclear models that appeared in this period is the LDM, which was used to explain the various aspects of nuclear phenomena, which are based on several assumptions, including that the nuclei are made of an incompressible material, that the nuclear force is identical for each nucleus, etc. [3].

Beautiful extensions of the concept of the semi-empirical mass formula SEMF were obtained by N. Bohr & Wheeler and was used to describe the phenomenon of nuclear fusion and find the nuclear binding energy. It depends partly on the theory and partly on the experimental calculations. Therefore, it is called the SEMF [4].

In addition to the LDM, another model appeared, the QLM. The Iranian scientist (Ghahramany) assumed that the atomic nuclei consist of a mixture of quarks - gluon instead of protons and neutrons. The strong nuclear force works weakly because of the considerable distance between quarks inside the nucleus, causing the formation of a mixture of quarks - gluons. Through it, he assumed the QLM, and he succeeded in calculating the nuclear binding energy [5].

Despite the successes of the LDM in explaining many phenomena such as fission and fusion processes and the existence of pairing, it failed to explain the appearance of magic nuclei, additional binding energy, and scaling of stability resulting from these magic numbers of nucleons. The QLM was able to explain the magic numbers, but it does not contain a specific term for these numbers.

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Therefore, the LDM and the QLM are incomplete structures. It is necessary to find a term for the shell, as it is expected to increase the theoretical binding energy of the magic nuclei and approach the experimental values [6].

Studying atoms showed that when the number of electrons is equal to one of the numbers (2, 10, 18, 36, 54, 86), it delivers high chemical stability and has a high ionization ability. According to the Pauli exclusion principle, these numbers, known as atomic magic numbers, can explain why these atoms' chemical stability is due to their closed electron shells and subshells [7]. Similarly, the nuclear binding energy of a nucleus whose number of protons, neutrons, or both is equal to one of the numbers (2, 8, 20, 28, 50, 82, 126) is high compared to the neighbouring nucleus. The nucleus that contains these numbers of nucleons is called the magic nucleus. These magic nuclei are not only very stable but also show other properties: their presence in a large proportion of nature and a large number of stable isotopes. The energy of separating protons and neutrons is considerable [8].

The researchers (Dong et al., 2010) [9] added two terms for closed shells when generalizing the LDM to clarify the effects of closed shells on determining the alpha decay energy of heavy nuclei. The researchers (Kalia et al., 2011)[10] developed the SEMF. They used a corrective term for the shell introduced by Segeer (Segeer, 1961)[11], a function of the number of nucleons. This term explains magic nuclei by predicting different parameters, for example, protons' separation energies and neutrons' separation energies. The researcher (Blake, 2011)[12] derived a new model for calculating the nuclear binding energy for light nuclei so that it is identical to the experimental values, as this region is filled with closed nuclear shells, which in turn givedifferent values from the experimental values. The new model relied on the bonding method by preparing the quarks (which contain protons and neutrons) and their relationship with the electromagnetic repulsion of protons to determine the nuclear binding energy of the twelve isotopes of deuterium in agreement with the experimental value with a value equal to (0.999). The researchers (Ghahramany et al., 2011) [5] proposed a new model to determine the nuclear binding energy for a wide range of light, medium, and heavy nuclei, depending on the number and mass of quarks. They called the model a QLM, and its results were compared with each experimental and LDM, noting an acceptable agreement. The researcher (Mavrodiev, 2016)[13] presented a generalization of the SEMF, which describes the measured nuclear mass values for 2654 nuclei in the AME2012 nuclear database with an accuracy of less than 2.2 MeV, starting with the number of Z = 1, and the number of N = 1, The effect of the magic numbers was determined, which are nine protons (2, 8, 14, 20, 28, 50, 82, 108, 124) and ten neutrons (2, 8, 14, 20, 28, 50, 82, 124, 152, 202).

This paper aims to add a corrective term to LDM and QLM, which is the term for closed shells of magic nuclei. This was done by relying on valence nucleons. In addition to fit of the volume, surface, coulomb repulsion, asymmetry, pairing and shell term in LDM by LSM. This was done by designing code in Fortran 95 software for 261 magic nuclei within the range ($2\leq Z\leq 92$), in order to correct and balance the nuclear binding energy values, especially for magic nuclei with the experimental values. A fit of the quark model has also been made in order to correct the values of the nuclear binding energy.

2. THEORETICAL FRAMEWORK

2.1 Liquid Drop Model (LDM)

The LDM, proposed by G. Gamow and developed by N. Bohr in 1939 [4], is one of the essential basic models proposed to calculate the binding energy in nuclear physics. Experiments revealed that the nuclei were essentially spherical bodies, with sizes that could be distinguished by radii proportional to $A^{1/3}$, indicating that the nuclear densities were almost independent of the nucleon number. Naturally, this leads to a model that envisions the nucleus as an incompressible liquid drop, in which nucleons play a role similar to the molecules in a regular liquid drop. In this picture, known as the LDM, the individual quantum properties of nucleons are entirely ignored. As in the case of a liquid drop, this model assumes that the nuclei have a specific "surface tension", where the nucleons behave similarly to those of molecules in the liquid [14]. The decay of nucleons by the emission of particles (such as alpha and beta particles) is similar to the evaporation of particles from a liquid's surface. Nucleons interact strongly with their nearest neighbours, as particles do in a liquid drop. Their properties can be described by the corresponding quantities, i.e., radius, density, surface tension, and volume energy [3].

We use the LDM to derive the SEMF, where the nucleus's binding energy or mass is expressed [15]:

$$B(A,Z) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_a \frac{(\frac{A}{2} - Z)^2}{A} \pm a_p A^{-1/2} \qquad (1)$$

where $(a_v, a_s, a_c, a_a, a_\rho)$ represents the term of volume, surface, coulomb, asymmetry, and pairing term, respectively.

2.2. Adding a Correction Term to the Shell by the valence nucleon

The effects of magic nuclei are one of the important topics in nuclear physics. Several ways to explain these effects have been suggested in the literature. However, there is no limit that handles reliable magic cores, which can form an additional term in the SEMF. The corrective term that we will adopt in this paper is the valence nucleon coefficient [16,17]:

$$B_{shell}(N_n, N_P) = a_{sh1}P + a_{sh2}P^2 \qquad (\mathbf{2})$$

where $P = (N_n N_p)/(N_n + N_p)$, and(N_n, N_p), are represent N and Z valence, which are located in the last energy levels and in turn participate in spinning the nucleus. When these two terms are added to the SEMF in an equation (1), they become as follows:

$$B(A,Z) = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z^2}{A^{\frac{1}{3}}} - a_a \frac{(\frac{A}{2} - Z)^2}{A} \pm a_p A^{-\frac{1}{2}} - a_{sh1}P + a_{sh2}P^2 \qquad (\mathbf{3})$$

All constants of the above equation are found using the least-squares method except for (a_{sh1}, a_{sh2}), since this model is for magic nuclei only, we will use the LSM to obtain constants specific to this model. Since the proposed shell terms are only for magic nuclei, we will code for magic nuclei to get new constants. That is, this model will only apply to magic nuclei.

And we will reduce the amount of error (ε) in calculating the coefficients (volume, surface, coulomb, asymmetry, pairing, and shell) of the equation (3) by taking 261 magic nuclei. The following equation represents the quantity (ε).

$$\mathcal{E} = \sum_i \left(y_i - B(Z_i, A_i)\right)^2 = \sum_i \left(y_i - Bi\left(a_v, a_s, a_c, a_a, a_\rho, a_{sh1}, a_{sh2}\right)\right)^2 \qquad (\mathbf{4})$$

where (y_i) is empirical value for the binding energy of the nucleus, and $B(Z_i, A_i)$) is the theoretical binding energy that we get from the equation (1). In other words, there is a possibility to obtain the coefficients of volume, surface, coulomb, asymmetry, and pairing by minimizing the function ($\mathcal{E}$). In other words, its first derivative must be equal to zero.

$$\frac{\partial \mathcal{E}}{\partial a_v} = -2\sum_i A_i\left(y_i - B(Z_i.A_i)\right) = 0$$

$$\frac{\partial \mathcal{E}}{\partial a_s} = 2\sum_i A_i^{\frac{2}{3}}\left(y_i - B(Z_i.A_i)\right) = 0$$

$$\frac{\partial \mathcal{E}}{\partial a_c} = 2\sum_i \frac{Z_i(Z_i-1)}{A_i^{\frac{1}{3}}}\left(y_i - B(Z_i.A_i)\right) = 0$$

$$\frac{\partial \mathcal{E}}{\partial a_a} = 2\sum_i \frac{(A_i - 2Z_i)^2}{A_i}\left(y_i - B(Z_i.A_i)\right) 0$$

$$\frac{\partial \mathcal{E}}{\partial a_\rho} = -2\sum_i A_i^{-\frac{1}{2}}\left(y_i - B(Z_i.A_i)\right) = 0$$

$$\frac{\partial \mathcal{E}}{\partial a_{sh1}} = 2\sum_i P_i\left(y_i - B(Z_i.A_i)\right) = 0$$

$$\frac{\partial \mathcal{E}}{\partial a_{sh2}} = -2\sum_i P^2{}_i\left(y_i - B(Z_i.A_i)\right) = 0 \qquad (\mathbf{5})$$

Through equation (5), we get the matrix equation (6).

$$\begin{bmatrix}
+\sum_i A_i^2 & -\sum_i A_i^{\frac{5}{3}} & -\sum_i {Z_i}^2 A_i^{\frac{2}{3}} & -\sum_i \left(\frac{Ai}{2} - Z_i\right)^2 & +\sum_i \boldsymbol{\delta} i\mathrm{A}i^{1/2} & -\sum_i A_i P_i & +\sum_i A_i Pi^2 \\
+\sum_i A_i^{\frac{5}{3}} & -\sum_i A_i^{\frac{4}{3}} & -\sum_i {Z_i}^2 A_i^{\frac{1}{3}} & -\sum_i \frac{\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{1}{3}}} & +\sum_i \boldsymbol{\delta} i\mathrm{A}i^{1/6} & -\sum_i A_i^{\frac{2}{3}} P_i & +\sum_i A_i^{\frac{2}{3}} Pi^2 \\
+\sum_i {Z_i}^2 A_i^{\frac{2}{3}} & -\sum_i {Z_i}^2 A_i^{\frac{1}{3}} & -\sum_i \frac{Z_i^4}{A_i^{\frac{2}{3}}} & -\sum_i \frac{{Z_i}^2\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{4}{3}}} & +\sum_i \frac{\boldsymbol{\delta} i{Z_i}^2}{A_i^{\frac{5}{6}}} & -\sum_i \frac{Z_i(Z_i-1)}{A_i^{\frac{1}{3}}} P_i & +\sum_i \frac{Z_i(Z_i-1)}{A_i^{\frac{1}{3}}} Pi^2 \\
+\sum_i \left(\frac{Ai}{2} - Z_i\right)^2 & -\sum_i \frac{\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{1}{3}}} & -\sum_i \frac{{Z_i}^2\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{4}{3}}} & -\sum_i \frac{\left(\frac{Ai}{2} - Z_i\right)^4}{A_i^2} & +\sum_i \frac{\boldsymbol{\delta} i\left(\frac{Ai}{2} - Z_i\right)^2}{A_i^{\frac{3}{2}}} & -\sum_i \frac{(A_i - 2Z_i)^2}{A_i} P_i & +\sum_i \frac{(A_i - 2Z_i)^2}{A_i} Pi^2 \\
+\sum_i \boldsymbol{\delta} i\mathrm{A}i^{1/2} & -\sum_i \boldsymbol{\delta} i\mathrm{A}i^{1/6} & -\sum_i \frac{\boldsymbol{\delta} i{Z_i}^2}{A_i^{\frac{5}{6}}} & -\sum_i \frac{\boldsymbol{\delta} i\left(\frac{A_i}{2} - Z_i\right)^2}{A_i^{\frac{3}{2}}} & +\sum_i \frac{\boldsymbol{\delta} i^2}{A_i} & -\sum_i A_i^{-\frac{1}{2}} P_i & +\sum_i A_i^{-\frac{1}{2}} Pi^2 \\
+\sum_i A_i P_i & -\sum_i A_i^{\frac{2}{3}} P_i & -\sum_i \frac{Z_i(Z_i-1)}{A_i^{\frac{1}{3}}} P_i & -\sum_i \frac{(A_i - 2Z_i)^2}{A_i} P_i & +\sum_i A_i^{-\frac{1}{2}} P_i & -\sum_i Pi^2 & +\sum_i Pi^3 \\
+\sum_i A_i Pi^2 & -\sum_i A_i^{\frac{2}{3}} Pi^2 & -\sum_i \frac{Z_i(Z_i-1)}{A_i^{\frac{1}{3}}} Pi^2 & -\sum_i \frac{(A_i - 2Z_i)^2}{A_i} Pi^2 & +\sum_i A_i^{-\frac{1}{2}} Pi^2 & -\sum_i Pi^3 & +\sum_i Pi^4
\end{bmatrix}$$

$$\begin{bmatrix} av \\ a_s \\ a_c \\ a_a \\ a_p \\ a_{sh1} \\ a_{sh2} \end{bmatrix} = \begin{bmatrix} \sum_i y_i A_i \\ \sum_i y_i A_i^{\frac{2}{3}} \\ \sum_i y_i \frac{Z_i^2}{A_i^{\frac{1}{3}}} \\ \sum_i y_i \frac{\left(\frac{Ai}{2}-Z_i\right)^2}{A_i} \\ \sum_i \frac{\delta i y_i}{A_i^{\frac{1}{2}}} \\ \sum_i P_i \\ \sum_i Pi^2 \end{bmatrix} \quad (6)$$

By designing a code to solve the above matrix equation by Gauss's method, we get new constants that are tabulated in the equation (7)[18]:

$$B(A,Z) = 14.2\,A - 15.3\,A^{\frac{2}{3}} - 0.57\,\frac{Z^2}{A^{\frac{1}{3}}} - 19.4\,\frac{(\frac{A}{2}-Z)^2}{A} \pm 12\,A^{-\frac{1}{2}} - 0.63\,P + 1.74\,P^2 \quad (7)$$

We will apply the above equation for magic nuclei only, once without the shell term and call it a model (LDM1), and once with the shell term, We will call it a Generalized LDM (GLDM). We will notice that the theoretical values are close to the experiment in the presence of the shell limit and with a very acceptable deviation rate.

2.4 Quark-Like Model

The QLM assumes that atomic nuclei are made up of a mixture of quark-gluon rather than protons and neutrons[19], and that the strong nuclear force works weakly because of the large distance between quarks inside the nucleus, causing the formation of the quark-clonal mixture. In order to calculate the nuclear binding energy, several assumptions were made as follows:

1. The nuclear binding energy is about 1% of the energy of the remaining mass that makes up the quarks, where ,$m_q c^2$ represents the mass of the up and down quarks.
2. The binding energy depends on a certain limit such as $(\frac{N^2-Z^2}{Z})$ due to the asymmetric distribution of the up and down quarks, and also because of the presence of the Coulomb force between them.
3. Nuclear binding energy depends on the size of the quark-colour mixture inside the nucleus. So it is proportional to 3A, instead of A, where A is the mass number.

The above assumptions led to the following formula for calculating the nuclear binding energy [20]:

$$B(A,Z) = [3A - \left(\frac{(N^2-Z^2)+\delta(N-Z)}{Z} + 3^2\right)] \times \frac{m_u c^2}{100} \quad (8)$$

Since:

$m_u c^2$: represents the mass of the up quark and its value is 330 Mev [21].

3^2: is the number of quarks that make up the nucleon.

$\delta(N-Z)$: is considered the nuclear beta stability line condition and its value is determined as follows:

$$\delta(N-Z) = \begin{cases} 0 & for\ N \neq Z \\ 1 & for\ N = Z \end{cases} \quad (9)$$

The mass of the top quark is shown in equation (8) because the top quark plays a major role in building the more stable baryon. That is, the proton. The coefficient (3^2) is explained by the fact that in order to form the lightest nuclei in this model, at least three or nine quarks must be involved. The surface limit is ignored in the quark model, because on the surface of the nucleus, the number of bonding quarks is small compared to those in the core of the nucleus, so the surface limit will be ignored [22].

2.5 FIT QLM

The QLM was assumed for calculating the theoretical nuclear binding energy values compared to their experimental values. The model succeeded in calculating the theoretical binding energy, but it does not give good results when compared with the experimental. So we will modify the model by making it fit the QLM. This fitting was done through a graph representing the experimental values of the nuclear binding energy on the y-axis with their theoretical values on the x-axis. The calibration equations were obtained through Figure (2), with a fit of no less than (0.99) as follows:

$$y = 0.9319X - 0.2561 \quad (10)$$

By substituting the x value into the calibration equation (8) with the equation (10), we get a generalization formula for calculating the nuclear binding energy as follows.

$$B(A,Z) = [3A - \left(\frac{(N^2 - Z^2) + \delta(N - Z)}{Z} + 3^2\right)] \times 3.07527 - 0.2561 \qquad (\mathbf{11})$$

And we will symbolize the above equation by the generalization QLM (GQLM).

Adding a Correction Term to the Shell by the valence nucleon

As with the LDM, we will work on adding the shell terms that depend on valence nucleons to the QLM. And represented by the equation (2) and using the same constants we found from the LSM. Thus, the QLM becomes as follows:

$$B(A,Z) = [3A - \left(\frac{(N^2 - Z^2) + \delta(N - Z)}{Z} + 3^2\right) - 0.63\,P + 1.74\,P^2\,] \times 3.07527 - 0.2561 \quad (\mathbf{12})$$

Applying the above equation to all the nuclei under study shows that the theoretical values are close to the experimental values with an acceptable deviation rate. It is worth noting that we will symbolize the above equation by the Generalization Quark–Like Model ($GQLM_1$).

2. 7 Determine the Standard Deviation of the Proposed Models

In order to determine the accuracy of the two equation (7),(12)and compare them with the experimental results, the standard deviation was calculated [23].

$$\sigma = \sum_{i=1}^{N} \frac{|\mathrm{BEexp} - \mathrm{BEtheo}|}{\mathrm{N}} \qquad (\mathbf{13})$$

BE_{exp} : representing experimental values.

BE_{theo}: representing theoretical values.

3 RESULTS AND DISCUSSION

Figures (1,2 and, 3) show a fit for the QLM

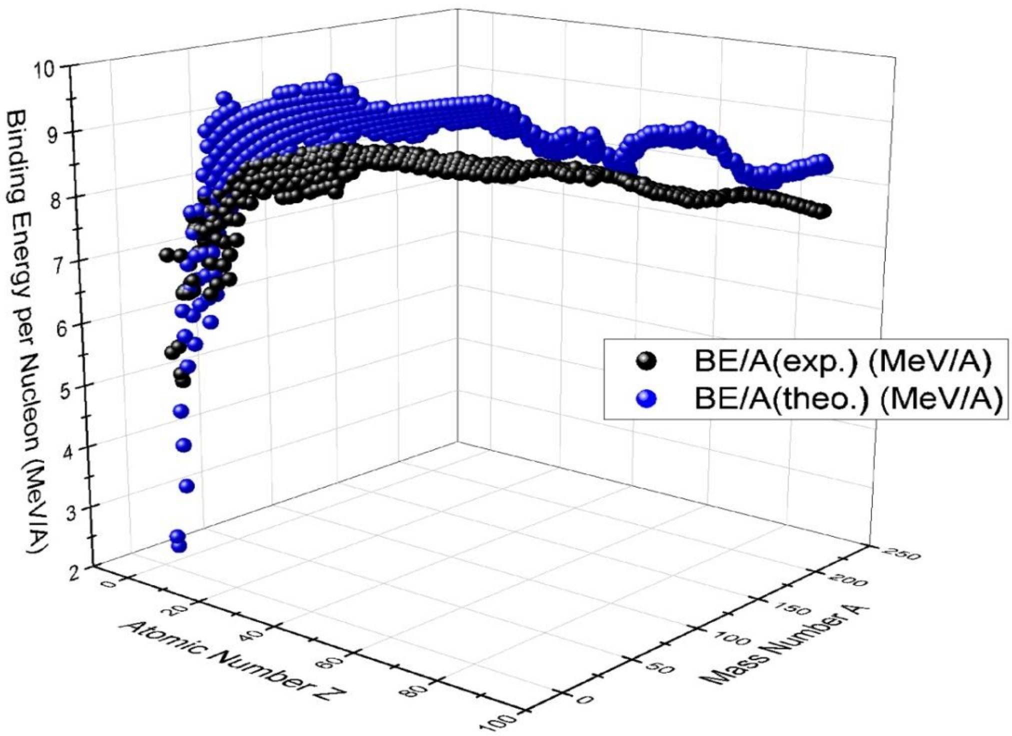

Fig. (1): Comparison between the average experimental binding energy and average theoretical binding energy that we obtained from equation (8) for all the studied nuclei.

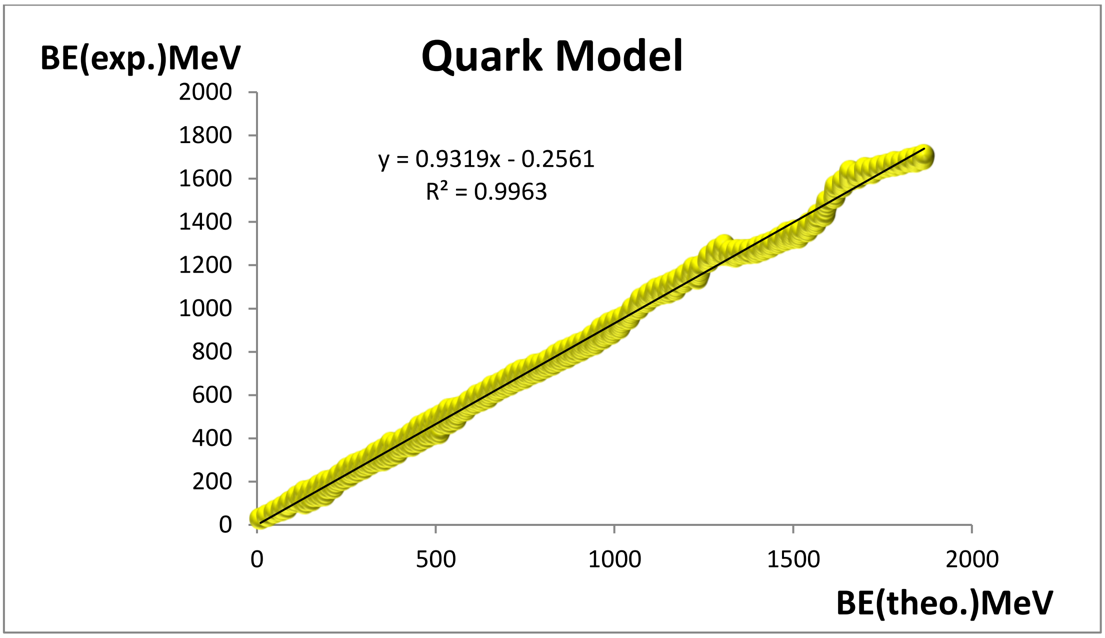

Fig. (2): The relationship between the experimental and theoretical nuclear binding energy according to the (QLM)

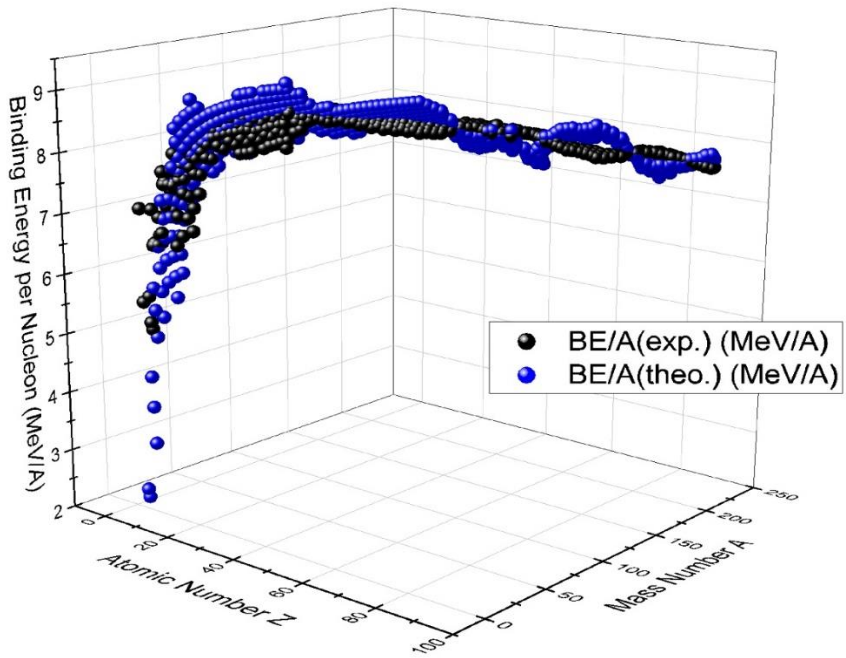

Fig. (3): The rate of the nuclear binding energy with the mass number (A) and the atomic number (Z) for the experimental and theoretical values of the model (GQLM).

It is evident from Figure (1) that the results we obtained from equation (8) show a significant discrepancy between the theoretical and experimental values. Therefore, after applying the fit on the nuclear binding energies of the QLM, represented by the Figure (2) we get the generalized quark-like model (GQLM) represented by equation (11), which showed results that are acceptable to the experimental values are shown in Figure (3).

Figures (4 and 5) show the relationship of the difference between the experimental binding energy and the theoretical binding energy in the presence of the shell-term with the mass number A and atomic number Z of models (GLDM) and (GQLM), for the studied nuclei within the range ($2 \leq Z \leq 92$) respectively.

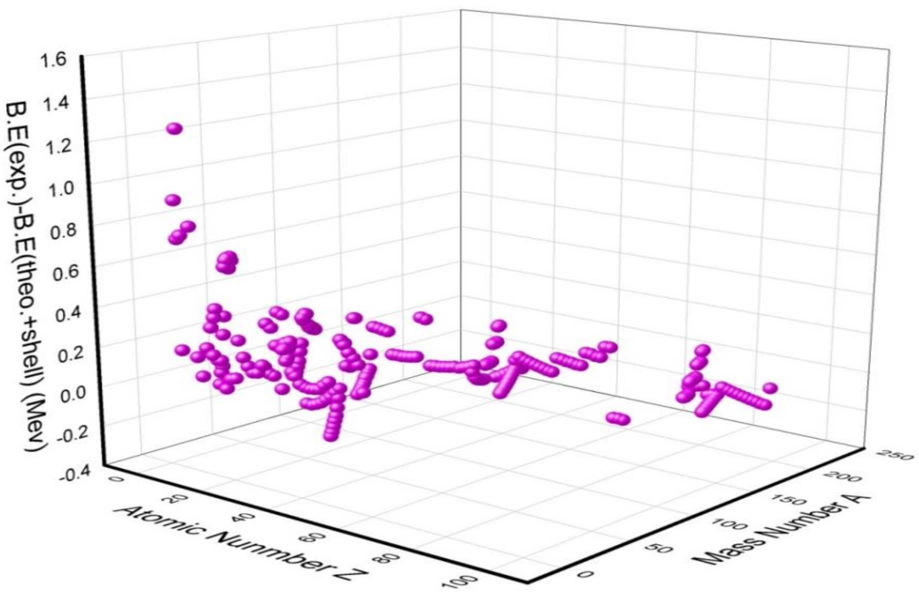

Fig. (4): The relationship of the difference between the experimental and theoretical binding energy in the presence of the shell-term with the mass number and atomic number according to the model (GLDM), for all studied magic nuclei.

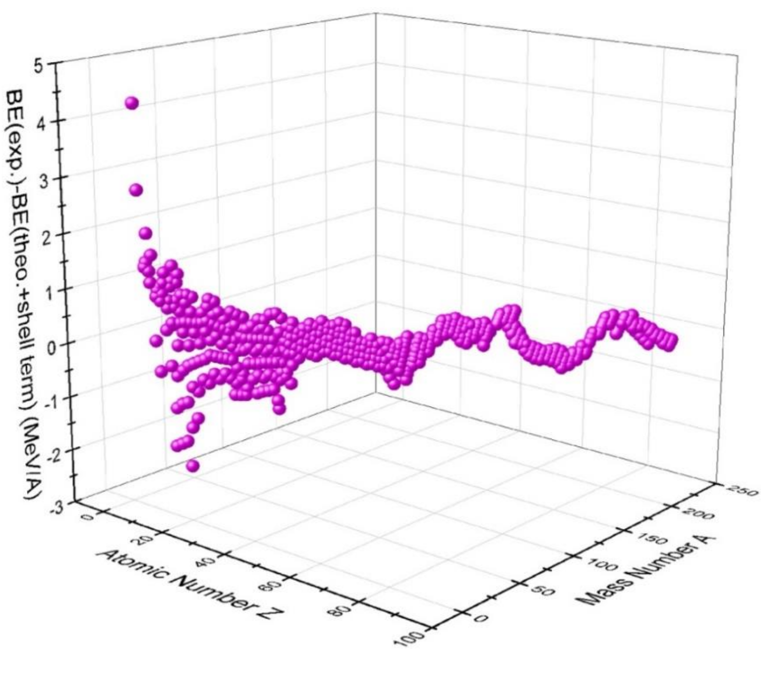

Fig. (5): the relationship of the difference between the experimental and theoretical binding energy in the presence of the shell term with the mass number and atomic number according to the model (GQLM) for all studied nuclei.

It is shown in Figure (4) of the model (GLDM), that the majority of the difference between experimental and theoretical nuclear binding energy with the shell term for magic nuclei is centered around zero. While the figure (5) and model (GQLM), shows that the difference between the experimental and theoretical nuclear binding energy with the presence of the shell term is of high value for the light nucleus of the magic numbers, whether in the number of neutrons or the number of protons, as well as the neighboring nuclei, especially for the nucleus (A = 4,5, 8,11), while this difference decreases significantly and is concentrated around zero in medium and heavy nuclei, which leads to acceptable agreement with experimental values. This indicates that the shell terms represented by valence nucleons can be added to the LDM and the QLM. It is clear to us by the standard deviation in Table (1) that the additive corrective term is true, and thus LDM and QLM become complementary. Especially to find new coefficients using LSM and make the QLM fit.

Table (1) The standard deviation values of the two models used before and after adding the shell term, along with the percentage of improvement that occurred in the two models.

The model	**Standard deviation(σ) *without shell***	**The model**	**Standard deviation(σ) *with shell***	**Improvement rate**
LDM	.0323	GLDM	0.144	**55%**
QLM	0.916	GQLM	0.84	**8%**

The standard deviation values indicate that the two proposed models can be adopted in the interpretation of magic numbers. The results of the model (GLDM) can be considered as very acceptable due to the improvement shown by the model (GLDM) of (55%) Compared with (17) which obtained an estimated improvement rate of (54%) using the same method, while the results of the model (GQLM) can be considered as somewhat acceptable because of the slight improvement that we obtained so the improvement rate is (8%).

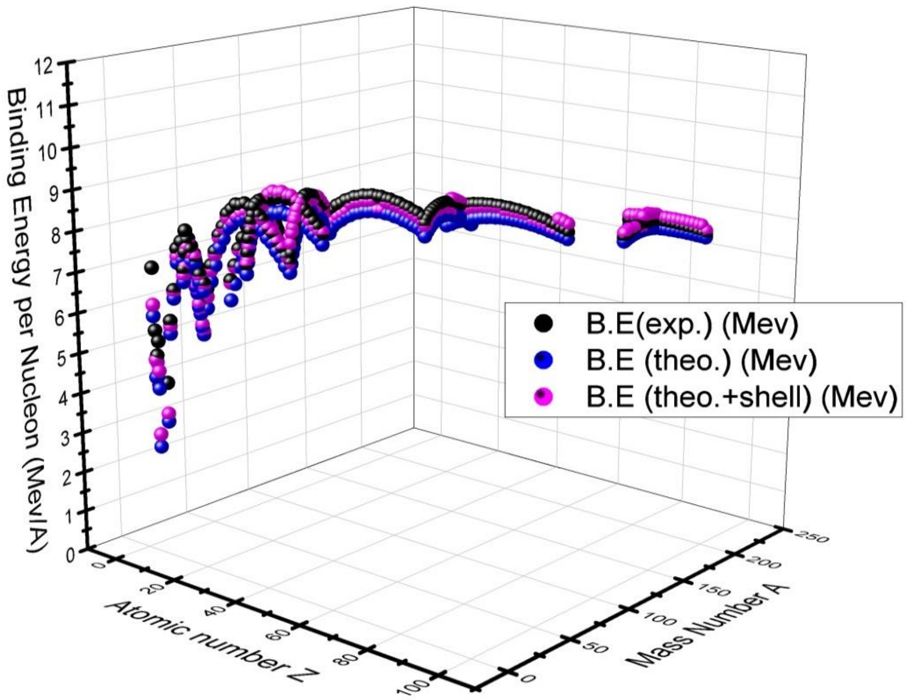

Fig. (6): the average nuclear binding energy with mass number A and atomic number Z for experimental and theoretical values of the original LDM and theoretical values of the models (GLDM) which contains magic nuclei.

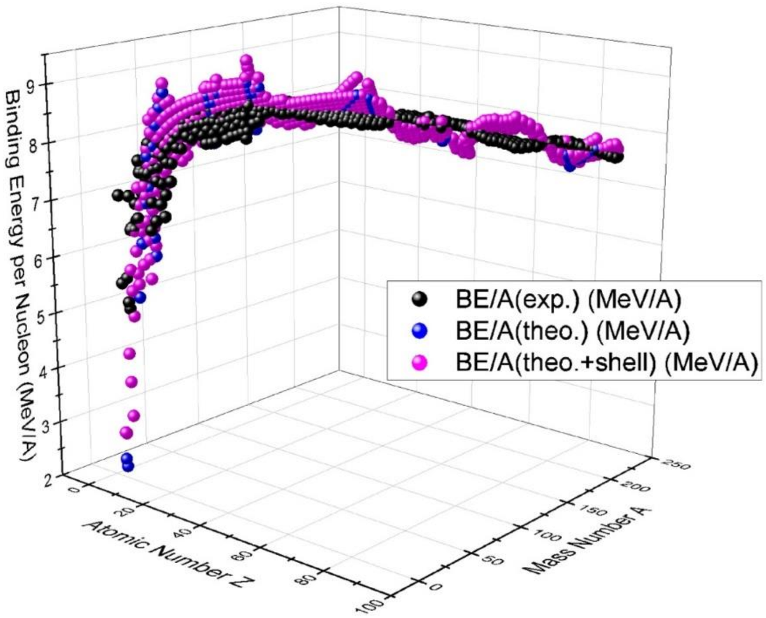

Fig. (7): The average nuclear binding energy with mass number A and atomic number Z for experimental and theoretical values of the original QLM and theoretical values of the models (GQLM) which contains magic and non-magic nuclei.

It can be seen from the two figures (6,7) that there is an acceptable agreement on the experimental nuclear binding energy rate with the theoretical values calculated through models (GLDM), (GQLM). We note that the theoretical values, after adding the shell term, are close to the experimental values of the models. This confirms the possibility of adopting the proposed shell term in the interpretation of magic numbers for all studied nuclei and for a wide range, especially for medium and heavy nuclei. We observed the actual increases in the binding energy for magic numbers, which led to the theoretical nuclear binding energy remarkably close to the experimental values, in addition to the standard deviation values and the improvement ratio as in Table (1) for the two proposed models which indicates the possibility of adopting the two models.

4. CONCLUSION

The effect of the term chance on LDM and QLM is very significant. From the results the following can be concluded:

- The results obtained through the proposed models (GLDM), ($GLDM_1$), in the interpretation of magic numbers, show that there is an acceptable agreement between the experimental values and the theoretical values calculated by us.
- The fit of the (GLDM) model as well as the QLM contributed to the improvement of the results
- The statistical relationships of the standard deviation showed the possibility of adopting the two models in interpreting of magic numbers.
- The results showed that the improvement ratio of the Model (GLDM), is higher compared to the Model (GQLM).

5. ACKNOWLEDGEMENT

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