Arab J. Nucl. Sci. Appl., Vol. 55, 2, 146-153 (2022)

ISSN 1110-0451

Arab Journal of Nuclear Sciences and Applications

Web site: ajnsa.journals.ekb.eg

(E S N S A)

The effect of power law and logarithmic of wind speed in calculation Diffusion from a Point Source

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ARTICLE INFO

Article history:

Received: 26th Sept. 2021

Accepted: 29th Dec. 2021

Keywords:

Urban Atmosphere;

Vertical Wind Shear;

Power Law;

Logarithmic law;

Stability Categories.

ABSTRACT

Investigation of a diffusion model of a substance from a point source in an urban area was carried out. The concentration (C) at the edge of the plume falls to almost zero for neutral, stable and unstable conditions. The new in this work is to consider the vertical wind speed (u) to be consisting of the sum of power and logarithmic law. This consideration was applied on the previous conditions. The calculated concentrations for these three conditions was compared with the observation's concentration data measured for Iodine-131 (I^{131}) at the reactor at Inshas. The emission rate (Q) was corrected (Q-corrected) from the observed results. In addition, the laps rate ($\Delta T/\Delta Z$) was found to be equals to (0.36), which is only in agreement with the stable case. Accordingly, the value of (Q) for this stable case was calculated and found to be greater than that calculated for each separated vertical wind speed. This result showed that the calculated ground concentration (C- corrected) is the same as that observed. This conclusion demonstrate that the suggested model is the best model suitable for this case.

1- INTRODUCTION

The advection-diffusion equation is solved with Gaussian model considering that the vertical eddy diffusivity and wind speed are invariant [1- 4]. The advection-diffusion equation is evaluated by taking the vertical eddy diffusivity and wind speed as functions of power law [5-7]. Also, studying the influence of eddy diffusivity and wind speed as functions of power law on the advection-diffusion equation was investigated [8]. Advection-diffusion equation in Two-dimensional is calculated by separation of variables, considering the vertical turbulence as a function of downwind distance and power law of vertical height and the wind speed as a function of power law [9-10]. Advection-diffusion equation with steady state in three-dimensional is solved using Fourier transform and considering vertical eddy diffusivity as a function of downwind distance and constant wind speed to obtain the normalized crosswind integrated concentration [11]. Also, the same problem is obtained by assuming that the vertical eddy diffusivity as function of power law of vertical height [12]. Lately, the answer of the advection–diffusion equation in two dimensions with variable vertical eddy diffusivity and wind speed is the use of Hankel rework is expected [13]. In addition, the solution of advection-diffusion equation in three dimensions using Hankel transform was obtained [14].

2- MATHEMATICAL MODEL

Fig. (1) shows the coordinate system direction of the mean wind. The effective height is H =h_s + Δh, where h_s is stack height and Δh is the plume rise which the plume travel downwind is increased where, the ground surface is a complete reflector of substance.

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DOI: 10.21608/ajnsa.2021.98037.1515

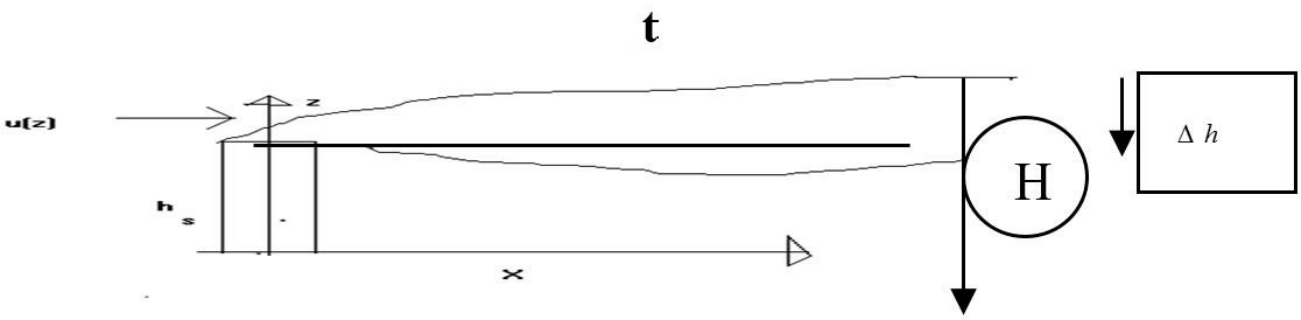

Fig. (A): Graph of pollutants diffusion of the trace.

The principle of mass conservative in steady state is as follows:

$$Q = \int_0^H \bar{u}(z) C(z) dz \qquad \textbf{(1)}$$

Where, Q is the emission rate, $\bar{u}(z)$ is the average wind speed, and C(z) is the concentration of material. To get the integration in Eq. (1) one can define $\bar{u}(z)$,H and C(z) as in the following sections.

2.1- Wind Profile

Power and Logarithmic wind law

The power and Logarithmic laws formulas are referred [15,1] as follows:

a- For Neutral Case:

$$u(z) = u_1\left(\frac{z}{10}\right)^n + \frac{u_*}{k} \ln\left(\frac{z+z_o}{z_o}\right) \qquad \textbf{(2)}$$

b- For Stable Case:

$$u(z) = u_1\left(\frac{z}{10}\right)^n + \frac{u_*}{k} \ln\left(\frac{z+z_o}{z_o} + \frac{5.2z}{L}\right) \qquad \textbf{(3)}$$

c- For Unstable Case:

$$u(z) = u_1\left(\frac{z}{10}\right)^n + \frac{u_*}{k}\left\{\ln\left[\frac{(1+f(z))^{\frac{1}{4}}-1}{(1+f(z))^{\frac{1}{4}}+1}\right] + 2\tan^{-1}(1+f(z))^{\frac{1}{4}} + \ln\left[\frac{(1+\frac{16z_o}{L})^{\frac{1}{4}}+1}{(1+\frac{16z_o}{L})^{\frac{1}{4}}-1}\right] + 2\tan^{-1}\left(1+\frac{16z_o}{L}\right)^{\frac{1}{4}}\right\} \qquad \textbf{(4)}$$

Where, z_0 is the roughness height (m), z_r=10m is the reference height, L is the Monim-Obukhov length, u_1 is the wind speed at reference height, u_* is the fraction velocity, k is Von-karman constant and $f(z)=16(z+z_o)$. The values of power-law exponent 'n' of air stability are referred [1] and presented in Table (1).

Table (1): Power-law exponent 'n' of eddy diffusivity as a function of air stability in urban area

	A	B	C	D	E	F
n	0.85	0.85	0.80	0.75	0.60	0.40

2.2- The effective height

The plume height Δh of diffusing substance at stack height h_s is calculated from the following equation [16]:

$$\Delta h = 3(w/u)D \qquad \textbf{(5)}$$

Where, the exit velocity is w (m/s), and D is the internal stack diameter (m). The effective stack height H equals:

$$H = h_s + \Delta h = h_s + 3(w/u)D \qquad \textbf{(6)}$$

2.3 Concentration Profile

The following partial differential equation

$$u\frac{\partial C}{\partial x} = \frac{\partial}{\partial z}\left(K(z)\frac{\partial C}{\partial z}\right) \qquad \textbf{(7)}$$

Was solved [21] under the following boundary conditions

$$C \to 0 \quad \text{and} \quad x \to \infty$$

$$K(z)\frac{\partial C}{\partial z} \to 0 \text{ as } z \to 0$$

The profile of concentration is assumed as follows:

$$C/C_0 = 1 + \alpha_1 (z/H) + \alpha_2 (z/H)^2 + \ldots \qquad \textbf{(8)}$$

where, C_o is the concentration value at the plume axis, C is the concentration value at a distance z away from the shaft axis, H is the effective stack height of the plume, and α_1 and α_2, etc are constants.

It was found that the series in Eq. (8) gives well fit to the observed data even if only the first two terms are retained as shown in Fig.(2) as follows:

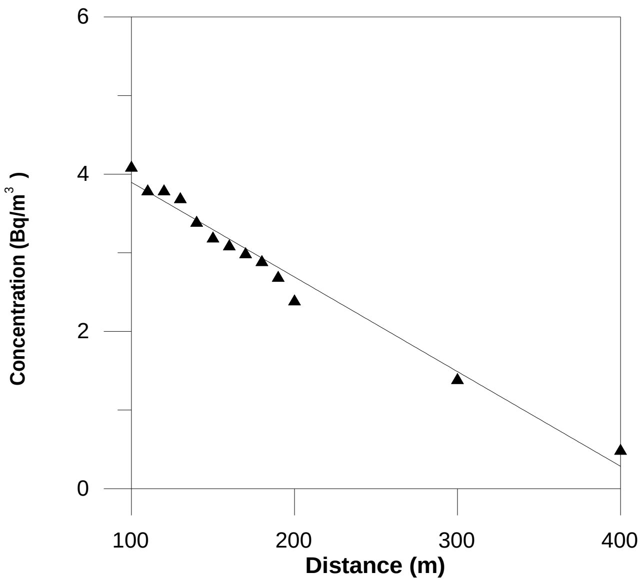

Fig. (2): The concentration of Iodine (I^{131}) via downwind distance from the reactor.

$$C/C_0 = 1 + \alpha_1 (z/H) \qquad \textbf{(9)}$$

Eq. (9) is a straight-line equation. The value α_1 depends on the concentration desired at the edge of the plume. If the edge of the plume is defined as having r percent of the concentration, then

$$\alpha_1 = -1 + 0.01\, r \qquad \textbf{(9a)}$$

and if "r = 0" then α_1= -1

$$C/C_o = 1 - (z/H) \qquad \textbf{(9b)}$$

2.4-Proposed Model

In Eq. (1) substituting $\bar{u}(z)$ by wind profiles which presented in Equations (2), (3) and (4), H by Eq. (7) and C(z) by concentration profile which presented in Eq. (9). One can get different forms during different stabilities as follows:

2.4.1 Neutral case

$$Q = \int_0^H \left(u_1 \left(\frac{z}{10} \right)^n + \frac{u_*}{k} \frac{\ln(z + z_o)}{z_o} \right) C_0 \left[1 + \alpha \left(\frac{z}{H} \right) \right] dz \qquad \textbf{(10)}$$

which, after integrating yields:

$$Q = \frac{u_1 C_0}{10^n} H^{n+1} \left(\frac{1}{n+1} + \frac{\alpha}{n+2} \right) + \frac{C_o u_*}{4kH} \left[\begin{array}{l} 2(H + z_o)(2H + \alpha H - \alpha z_o) \ln \dfrac{H + z_o}{z_o} \\ - H(4H + \alpha H - 2\alpha z_o) \end{array} \right] \qquad \textbf{(11)}$$

where, C_o is the concentration at the plume axis. Q is the emission rate. α_1= -1 in concentration profile, (Eq.9).

2.4.2 Stable case

$$Q = \int_0^H \left(u_1 \left(\frac{z}{10} \right)^n + \left(\frac{u_*}{k} \frac{\ln(z + z_o)}{z_o} + \frac{5.2z}{L} \right) \right) C_0 \left[1 + \alpha \left(\frac{z}{H} \right) \right] dz \qquad \textbf{(12)}$$

which, after integrating yields:

$$Q = \frac{u_1 H^{n+1} C_0}{10^n} \left(\frac{1}{n+1} + \frac{\alpha}{n+2} \right) + \frac{u_* C_o}{kH} \left[\frac{1}{4H} \left(\begin{array}{l} 2(H + z_o)(2H + \alpha H - \alpha z_o) \ln \dfrac{H + z_o}{z_o} \\ - H(4H + \alpha H - 2\alpha z_o \end{array} \right) + \frac{5.2H^2}{L} \left(\frac{1}{2} + \frac{\alpha}{3} \right) \right] \qquad \textbf{(13)}$$

2.4.3 Unstable case

Similarity, the conservative mass of the plume is as follows:

$$Q = \frac{u_1 H^{n+1} C_0}{10^n} \left(\frac{1}{n+1} + \frac{\alpha}{n+2} \right) + \frac{u_* C_0}{k} \Omega \qquad \textbf{(14)}$$

where, Ω is taken from [17].

2.5-Case study

The derived expression for C_o/Q is applying on of the first Research Reactor in Egyptian Atomic Energy Authority. The total ventilation rate is 39965 m^3/hr [18], which was emitted from the reactor stack of 43 m height, 1 m internal diameter, and exist velocity is 4 m/s, taking α=-1.

The predicted values of u, u_*, Δh, H and C_o/Q of neutral, stable and unstable conditions are presented in Table (2), (3) and (4) respectively. The last three columns in the three tables show that the usual continuous operation time of the reactor through 48 hours.

Table (2): The wind speed (u), friction velocity (u*), the plume rise (Δh), effective height (H) and the concentration at the axis of the plume over emission rate (C_o/Q) during the year 1999 in neutral case.

u (m/s)	u* (m/s)	Δh(m)	H (m)	C_o/Q *10^3 s/m^3 (ref. 1)	C_o/Q *10^3 s/m^3(ref. 2)	Proposed model C_o/Q *10^3 s/m^3
5.27	0.33	2.28	45.28	33.62	8.17	41.79
5.31	0.33	2.26	45.26	33.64	8.11	41.75
5.34	0.34	2.25	45.25	32.67	8.07	40.74
6.37	0.4	1.88	44.88	28.10	8.83	36.93
5.17	0.32	2.32	45.32	34.62	8.32	42.94
4.45	0.28	2.70	45.70	39.09	9.57	48.66
5.1	0.32	2.35	45.35	34.59	8.43	43.02
4.81	0.3	2.49	45.49	36.73	8.9	45.63
5.3	0.33	2.26	45.26	33.64	8.13	41.77
4.86	0.31	2.47	45.47	35.57	8.8	44.37
5.36	0.34	2.24	45.24	32.67	8.04	40.71
5.19	0.33	2.31	45.31	33.58	8.29	41.87
5.41	0.34	2.22	45.22	32.70	7.97	40.67
5.54	0.35	2.17	45.17	31.82	7.79	39.61
5.2	0.33	2.31	45.31	33.59	7.27	40.86
5.61	0.35	2.14	45.14	31.84	7.7	39.54
5.79	0.36	2.07	45.07	31.03	7.48	38.51
6.27	0.39	1.91	44.91	28.79	6.94	35.73
5.93	0.37	2.02	45.02	30.24	7.31	37.55
6.01	0.38	2.00	45.00	29.47	7.22	36.69
5.41	0.34	2.22	45.22	32.70	7.97	40.67
5.75	0.36	2.09	45.09	31.01	7.53	38.54
5.26	0.33	2.28	45.28	33.62	8.19	41.81

Ref. (1) is Essa and Maha (2006) [17] and Ref. (2) is Essa and Ghobrial (2013) [19].

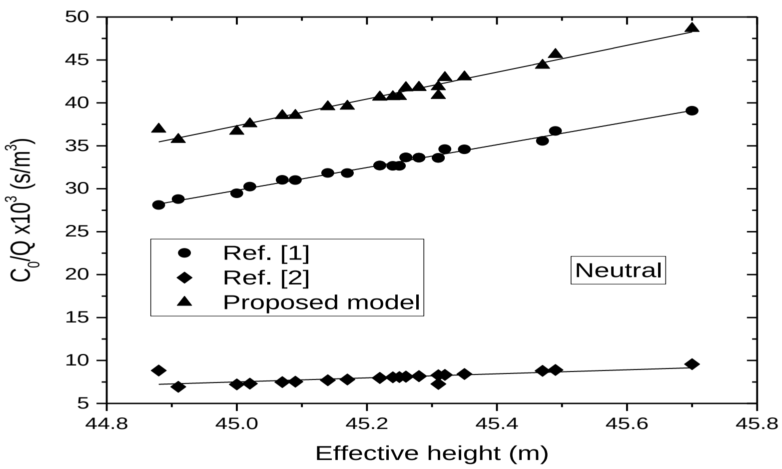

Fig. (3): The normalized concentration at the plume axis via the effective height in neutral condition

Table (3): The wind speed (u), friction velocity (u*), the plume rise (Δh), effective height (H) and the concentration at the axis of the plume over emission rate (C_o/Q) during the year 1999 in stable caseat L=55m.

u (m/s)	u* (m/s)	Δh(m)	H (m)	$C_o/Q *10^3$ s/m^3 (ref. 1)	$C_o/Q *10^3$ s/m^3(ref. 2)	Proposed model $C_o/Q *10^3$ s/m^3
5.27	0.33	2.28	45.28	17.99	8.7	26.69
5.31	0.33	2.26	45.26	20.97	9.93	30.9
5.34	0.34	2.25	45.25	19.64	9.5	29.14
6.37	0.4	1.88	44.88	16.62	7.87	24.49
5.17	0.32	2.32	45.32	20.90	10.19	31.09
4.45	0.28	2.70	45.70	21.61	10.52	32.13
5.1	0.32	2.35	45.35	21.66	10.34	32
4.81	0.3	2.49	45.49	22.38	10.85	33.23
5.3	0.33	2.26	45.26	21.63	10.45	32.08
4.86	0.31	2.47	45.47	20.97	9.95	30.92
5.36	0.34	2.24	45.24	19.63	9.52	29.15
5.19	0.33	2.31	45.31	20.27	9.75	30.02
5.41	0.34	2.22	45.22	20.94	10.07	31.01
5.54	0.35	2.17	45.17	19.63	9.54	29.17
5.2	0.33	2.31	45.31	22.39	10.79	33.18
5.61	0.35	2.14	45.14	19.67	9.37	29.04
5.79	0.36	2.07	45.07	18.51	8.9	27.41
6.27	0.39	1.91	44.91	18.52	8.88	27.4
5.93	0.37	2.02	45.02	19.65	9.46	29.11
6.01	0.38	2.00	45.00	19.04	9.27	28.31
5.41	0.34	2.22	45.22	20.30	9.63	29.93
5.75	0.36	2.09	45.09	20.26	9.81	30.07
5.26	0.33	2.28	45.28	21.68	10.27	31.95

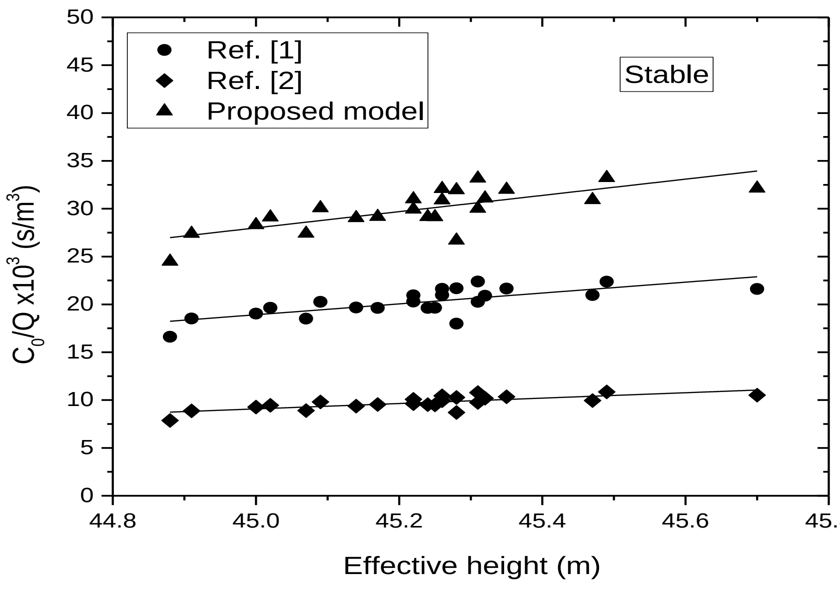

Fig. (4): The normalized concentration at the plume axis via the effective height in stable condition

Table (4): The wind speed (u), friction velocity (u*), the plume rise (Δh), effective height (H) and the concentration at the axis of the plume over emission rate (C_o/Q) during the year 1999 in unstable class at L=2.5m.

u(m/s)	u* (m/s)	Δh(m)	H (m)	C_o/Q *10 ^{3} s/m^3 (ref. 1)	C_o/Q *10 ^{3} s/m^3(ref. 2)	Proposed model C_o/Q *10 ^{3} s/m^3
5.27	0.33	2.28	45.28	73.97	9.68	83.65
5.31	0.33	2.26	45.26	76.15	11.13	87.28
5.34	0.34	2.25	45.25	75.80	10.65	86.45
6.37	0.4	1.88	44.88	72.22	8.78	81
5.17	0.32	2.32	45.32	76.60	11.44	88.04
4.45	0.28	2.70	45.70	77.64	11.82	89.46
5.1	0.32	2.35	45.35	77.13	11.61	88.74
4.81	0.3	2.49	45.49	78.07	12.19	90.26
5.3	0.33	2.26	45.26	77.07	11.73	88.8
4.86	0.31	2.47	45.47	76.74	11.16	87.9
5.36	0.34	2.24	45.24	75.79	10.67	86.46
5.19	0.33	2.31	45.31	76.26	10.1	86.36
5.41	0.34	2.22	45.22	76.67	11.3	87.97
5.54	0.35	2.17	45.17	75.78	10.69	86.47
5.2	0.33	2.31	45.31	77.48	12.13	89.61
5.61	0.35	2.14	45.14	75.29	10.24	85.53
5.79	0.36	2.07	45.07	74.40	9.96	84.36
6.27	0.39	1.91	44.91	74.41	9.94	84.35
5.93	0.37	2.02	45.02	75.82	10.6	86.42
6.01	0.38	2.00	45.00	75.34	10.38	85.72
5.41	0.34	2.22	45.22	75.73	10.79	86.52
5.75	0.36	2.09	45.09	76.22	11.0	87.22
5.26	0.33	2.28	45.28	77.17	11.52	88.69

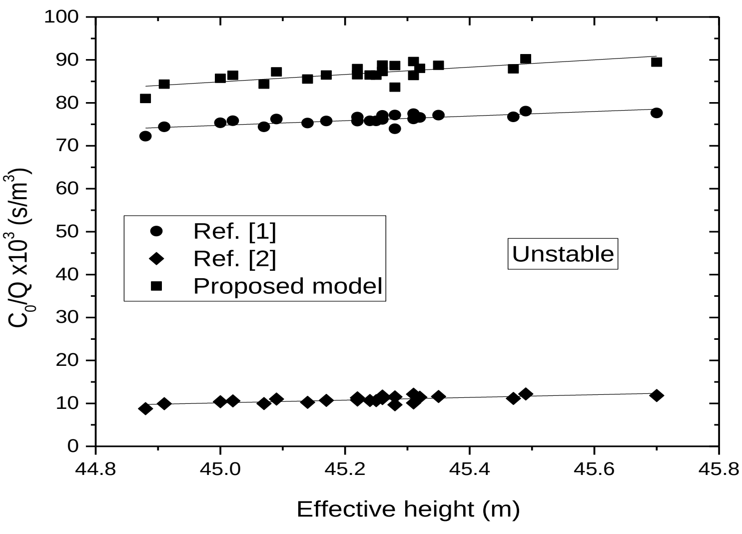

Fig. (5): The normalized concentration via the effective height in unstable condition

Figs (3-5) show that a straight-line fit well to this data in the case of neutral, stable and unstable conditions between the normalized concentration at the plume axis C_o/Q and effective height H. One finds that the data of the proposed model are larger than the power and Logarithmic laws. The smallest and moderate values are obtained in power and logarithmic laws for the wind speed respectively.

3. Verification

For a point source with h_s=27m (height of the source of the Second Research Reactor in Egyptian Atomic Energy Authority (ETRR-2) from the ground. For Iodine-131 (I^{131}), (H) is 31.29m, (Q) is 35 Bq, the wind speed (u_1) is 2.8 m/s and the lapse rate ($\Delta T/\Delta Z$) (C°/100m) is 0.36. This is **stable** case (n=0.5). from Eq. (13), one gets the concentration at (C_o) equals 0.822Bq/m^3. Then the concentration at ground modifies to:

$$C(ground) = 0.822\,(1 - \frac{h_s}{H}) = 0.113\,Bq/m^3 \quad \textbf{(15)}$$

The observed concentration at x=300m, H=31.29 m was 0.16 Bq/m^3. To achieve the verification, the source strength is adjusted to yield observed concentration at the first point of observation to get the corrected source strength as follows:

$$Q(corrected) = \frac{0.16 \times 35}{0.113} = 49.7$$

By using Q (corrected), one gets C_o (corrected)=1.17 and C_{ground} (corrected proposed model) =0.16Bq/m^3. It is a same as observed model. i e. The proposed model is a well perfect model because the ground corrected concentration (0.16Bq/m^3) is the same as the observed concentration (0.16Bq/m^3) than the previous ground concentrations in ref. (1) (0.18Bq/m^3) [17] and in ref. (2) (0.155Bq/m^3) [19].

4- CONCLUSIONS

The proposed model described the pollutant concentrations at point source which emits pollutant into the atmosphere, the results of this study describe:

(1) The mean wind speed should be as a constant quantity with height. In this study the sum of logarithmic and power law suggested by the authors has been used to get three different formulas for neutral, stable and unstable classes respectively, the model is being extended to the area and line source configurations.

(2) The two factors L and z_o are important factors in determining pollution concentrations. The model proposed in this study accounts for these parameters. One calculates the plume rise, effective height and the normalized concentration at the axis of the plume at the reactor release through different stability classes.

Also, we get the concentration at the ground of the Iodine-131 (I^{131}) which is the same as the observed concentration value and adjusted its source strength when the wind speed is the sum of logarithmic and power law than previous models.

The proposed model is a well perfect model because the ground corrected concentration (0.16Bq/m^3) is the same as the observed concentration (0.16Bq/m^3) than the previous ground concentrations in ref. (1) (0.18Bq/m^3) [17] and in ref. (2) (0.155Bq/m^3) [19]. It is concluded that the proposed model is more suitable than the two previous models.

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