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Mass Spectrum of exotic Hyperatoms

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ABSTRACT

The hyperatoms belong to the group of exotic atoms. An exotic hyperatom is a system where a negative muon as an electron in a normal atom is captured into an hyperatomic orbit around hypernuclei. Our knowledge of the non-relativistic quantum mechanics is a good method for the exotic hyperatoms containing one muon. We theoretically have been investigated exotic muonic hyperatoms. Theoretically, the mass spectra of muonic hyperatoms have been determined in the framework of the non-relativistic Schrödinger equation with Coulomb type potential describing the interaction between a muon and the hypernuclei core. The studies of exotic systems have achieved great progress both in theory and experiments. Mass spectrum and binding energy have been provided useful information for exotic hadronic systems and for hypernuclei which participate in the experimental reaction and also the interaction between them. The analytic method based on quantum field theories, the behavior of the correlation function, Feynman path integral, and oscillator representation method is suggested. Within all of these different frameworks, we have presented the theoretical approach to describing the mass spectrum of muonic hyperatoms in the ground and orbital excited states.

INTRODUCTION

Determination of hyperatoms masses by exact mass formula has been a very important issue in theoretical nuclear physics. This article proposes a well-known method called the oscillator representation method (ORM), which determines the bound state mass and constituent masses of hypernuclei and muon. The mass spectrum of the exotic atom as a bound state can be determined within the quantum field theory due to particle charge. The scalar charged particles current is $J(x)=\Psi^{+}(x)\Psi^{-}(x)$, for defining the bound state mass based on correlator $\Pi(x)$ which can be described by Green's function $G_m(x_i,x_j\,|\,E(x))$, all annihilation channels are neglected and Green's function product of scalar particles with masses m_1,m_2 in the external electrostatic field will be given as correlator $\Pi(x)_{|x=x_1-x_2|}=\left\langle G_{m_1}(x_1,x_2\,|\,E)\cdot G^{*}_{m_2}(x_2,x_1\,|\,E)\right\rangle$. The Green's function is presented in the functional integral form [2-4] $(\hbar=c=1)$

$$\langle J(x_1)J(x_2)\rangle_{E(x)}=\langle \Psi^{+}(x_1)\Psi^{-}(x_2)\Psi^{+}(x_2)\Psi^{-}(x_1)\rangle_{E(x)}$$
$$\langle \Psi(x_1)\Psi(x_2)\rangle_{E(x)}=\int\frac{dk}{2\pi}\widetilde{D}(k^2)e^{-ik(x_1-x_2)} \qquad \textbf{(1)}$$
$$G_m(x_1,x_2|E(x))$$
$$=\int_0^{\infty}\frac{du}{(4\pi u)^2}e^{-um^2-\frac{x^2}{4u}}\times\int NB\delta(x)\,e^{-0.5\int_0^1\dot{B}^2(\eta)d\eta+ig\int_0^S A(\eta)\frac{\partial Z(\eta)}{\partial\eta}d\eta}$$

In quantum field theory the bound state mass is defined with this equation $\Pi(x)_{|x=x_1-x_2|\to\infty}\approx e^{-M|x|}$, then after averaging over the external electrostatic field $E(x)$ one also finds a correlator on functional integral form [1]

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$$\Pi(x)_{|x|\to\infty} = \int_0^\infty\int_0^\infty \frac{d\mu_1 d\mu_2}{(8(x)\pi^2)^2} J(\mu_1,\mu_2)\, e^{-\frac{|x|}{2}\left(\frac{m_1^2}{\mu_1}+\frac{m_2^2}{\mu_2}+\mu_1+\mu_2\right)} \quad (2)$$

$J(\mu_1,\mu_2)-$ contains potential and nonpotential interactions. This article will not focus on relativistic correction for potential interactions, because the aim is to determine the mass spectrum and the related mass of the system. But in the asymptotic limit [1, 3]

$$\lim_{|x|\to\infty} J(\mu_1,\mu_2) = e^{-(x)E_\ell(\mu_1,\mu_2)} \quad (3)$$

Where function $E_\ell(\mu_1,\mu_2)=E_\ell(\mu)$ is the eigenvalue of the interaction Hamiltonian:

$$\hat{H}\Psi(x) = E_\ell(\mu_1,\mu_2)\Psi(x) \Rightarrow \hat{H} = \sqrt{\hat{p}^2+m_1^2} + \sqrt{\hat{p}^2+m_2^2} + V(x) \quad (4)$$

Then the bound state of hyperatom mass is defined as a minimum of relation

$$M = \frac{1}{2}\min_{\mu_1,\mu_2}\left[\frac{m_1^2}{\mu_1}+\frac{m_2^2}{\mu_2}+\mu_1+\mu_2+2E_\ell(\mu)\right] = \sqrt{m_1^2-2\mu^2E_\ell^{'}(\mu)}+\sqrt{m_2^2-2\mu^2E_\ell^{'}(\mu)}+\mu E_\ell^{'}(\mu)+E_\ell(\mu) \quad (5)$$

$$\mu_1=\sqrt{m_1^2-2\mu^2E_\ell^{'}(\mu)},\quad \mu_2=\sqrt{m_2^2-2\mu^2E_\ell^{'}(\mu)},\quad E_\ell^{'}(\mu)=\frac{\partial E_\ell(\mu)}{\partial\mu}$$

μ_1,μ_2- are the constituent mass of hypernuclei and muon in the bound state. m_1,m_2- are their rest mass, and $\mu=\dfrac{\mu_1\mu_2}{\mu_1+\mu_2}-$ is the bound state mass appearing in the n-body systems (the reduced mass). The binding energy for hyperatom like the normal atoms is calculated by (6) if the recoil effect is neglected i.e.:

$$m_1=\infty,\quad E_{bin}=M-m_1,\quad E_{bin}=\sqrt{m_2^2-2\mu^2E_\ell^{'}(\mu)}+\mu E_\ell^{'}(\mu)+E_\ell(\mu), \quad (6)$$

Hydrogen-muonic hyperatom

The interdiction section presented the bound state mass of two body systems based on the asymptotic behavior of the correlation functions of the electrostatic field currents. In addition, it presented a theoretical method for determining the bound state mass of hydrogen-muonic hyperatoms consisting of ${}^{x}_{\Lambda}H$ and μ^- with relativistic mass correction [4]. The dependence of the constituent mass of hypernuclei and muon on the current mass is analytically derived. Now the parameters should be determined for the calculation of the masses. Therefore, based on basic elements of the oscillator representation method (ORM), the hydrogen-muonic hyperatom mass spectrum can be determined by applying the transformation into the new 4D momentum space [3]. In quantum field theory, for the ground and excited states, the systems are described by an infinite number of oscillators that keep their oscillating characteristic in the interactions. To describe the mechanism of interaction and creation of exotic hyperatom, the Schrödinger equation is generally invoked based on the Gaussian solution for large distances. Therefore, we are presented the radial Schrödinger equations that is applied for spherically symmetric Coulomb potential ($V(x)=\alpha_S x^{-1}$) between the clusters (i.e., the hypernuclear core and the muon) and reads as follows[3]:

$$\hat{H}R(x)=E_\ell(\mu)R(x)\Rightarrow\left(\frac{\hat{p}^2}{2\mu}+\alpha_S x^{-1}\right)R(x) = E_\ell(\mu)R(x), \quad (5) \quad (7)$$

and to use quantum field methods of scalar field (the Gaussian solution), the variables ($x\to x(q)$)in the equation (7) have been changed by the following substitution:

$$x=x(q)=q^2\Rightarrow\Phi_\ell(x)\to q^{2\ell}\phi_\ell(q^2) \quad (8)$$

and the radial Laplacian operator reads $\Delta=\frac{d^2}{dx^2}+\frac{m-1}{x}\frac{d}{dx}$, and the Schrödinger equation (7) reads:

$$\left(-\frac{1}{2x}\left(\frac{d}{dx}\right)^2 x+\frac{\ell(\ell+1)}{2x^2}\frac{d}{dx}+V(x)\right)R(x)=E_\ell(\mu)R(x) \quad (9)$$

then the radial Laplacian operator in the new 4D momentum space is written in the following form

$$\hat{p}_x^2\to\frac{1}{4q^2}\hat{p}_q^2,\quad \nabla_x^2\to\nabla_q^2=\frac{d^2}{dq^2}+\frac{3+4\ell}{q}\frac{d}{dq} \quad (10)$$

after standard simplifications, we have obtained the modified radial Schrödinger equations, in the Coulombic potential where the wave function becomes an oscillator one. The modified Hamiltonian $\hat{H}\phi_\ell(q)=E_\ell(\mu)\phi_\ell(q)$ with Gaussian asymptotic of wave functions of hyperatom interactions has a start point to describe energy and masses. Based on ORM and $(\hat{a}^+,\hat{a}^-)$ operators, the canonical variables $(\hat{q},\hat{p}_q)$ in the (10) should be rewritten as:

$$\hat{a}^+=\sqrt{\frac{\omega}{2}}\left(\hat{x}-\frac{i}{\omega}\hat{p}\right),\quad \hat{a}^-=\sqrt{\frac{\omega}{2}}\left(\hat{x}+\frac{i}{\omega}\hat{p}\right),\quad \hat{x}=\frac{\hat{a}^-+\hat{a}^+}{\sqrt{2\omega}},\quad \hat{p}=\sqrt{\frac{\omega}{2}}\frac{\hat{a}^--\hat{a}^+}{i} \qquad \textbf{(11)}$$

and then it is necessary to extract the exact pure oscillator segment that has the frequency ω and rewrite Hamiltonian in the normal form of operators $\hat{a}^+,\hat{a}^-$ based on (10). In this way, we have defined

$$q^2=\frac{2(1+\ell)}{\omega_\ell},\quad p^2=2(1+\ell)\omega_\ell \qquad \textbf{(12)}$$

for more details see [3]. Then by definition, the free Hamiltonian of a quantum system $(\varepsilon_0(\omega))$ that oscillates with ω in the Coulomb potential interactions is obtained the new 4D momentum space [3-7]

$$\hat{H}_\ell\phi_\ell(q)=E_\ell(\mu)\phi_\ell(q)$$
$$\Rightarrow\left(\frac{\hat{p}_q^2}{2\mu}+4q^2\big(V(q)-E_\ell(\mu)\big)\right)\phi_\ell(q)=0\Rightarrow\varepsilon_0(\omega)\phi_\ell(q)=0 \qquad \textbf{(13)}$$
$$\varepsilon_0(\omega)=(1+\ell)\omega+4\mu\alpha_s q^2\left(-\frac{\alpha_s}{q^2}-E_\ell(\mu)\right)=0$$

ORM has required that the interaction segment of the Hamiltonian be free of quadratic (q^2) because by theory and based on essential constraints on ORM, this part lies in free oscillator segments. This condition provides an equation for free oscillator frequency ω_ℓ (i.e. $\varepsilon_0(\omega)$ is free of quadratic terms (q^2))[3]. The oscillator frequency ω, which determines the main quantum contribution in the next equations. Calculation of energy eigenvalue $E_\ell(\mu)$ is defined from ORM conditions with H_ℓ − Hamiltonian in the zeroth approximation [8]

$$\left(\varepsilon_0(\omega_\ell)=0,\quad \frac{\partial\varepsilon_0(\omega_\ell)}{\partial\omega_\ell}=0\right)\Rightarrow\left(E_\ell(\mu)=\frac{1}{8\mu}\omega_\ell^2-\frac{\alpha_s\Gamma(2+2\ell)}{\Gamma(3+2\ell)}\omega_\ell,\quad \omega_\ell=\frac{4\mu\alpha_s\Gamma(2+2\ell)}{\Gamma(3+2\ell)}\right) \qquad \textbf{(14)}$$

Then it will be possible to determine the mass spectrum of hyperatom in the ground state [8-11] and with orbital excitation, $\ell=1$.

RESULTS AND DISCUSSION

The ORM is a projective unitary representation of the symplectic group or oscillator explanation that we have determined parameters for the ground and excited states with the well-known potential interaction. The oscillator method is the representation of the canonical variables in the normal form (also called Wick order) and in the auxiliary space. Taking into account this representation we have obtained the expression for the ground and excited energy for the interaction Hamiltonian. We have defined the approach for predicting the mass spectrum of exotic hyperatom, based totally on the relativistic-quantum model. Two body exotic muonic hyperatom mass spectrum has been expressed as

$$M=\sqrt{m_{hypernuclei}^2-2\mu^2E_\ell'(\mu)}+\sqrt{m_\mu^2-2\mu^2E_\ell'(\mu)}+\mu E_\ell'(\mu)+E_\ell(\mu) \qquad \textbf{(15)}$$

and binding energy as a normal atom described by

$$E_{bin}=M-m_{hypernuclei} \qquad \textbf{(16)}$$

In the ORM approach, we have calculated the mass spectra of the exotic muonic hydrogen hypernuclei-bound state. The results of numerical calculation of all parameters that have mass dimension are presented with different constant interactions from coulomb $\alpha_s=0.007$ to semi-strong interaction $\alpha_s=0.5$. The ground and excited states bound state mass of two body hyperatom systems are determined from equations (15) and (16). The numerical results are shown in Tables (1) and Tables (2). with the rest values of particles in hydrogen hyperatoms (all mass values are given in (MeV)):

$m_n = 939.565421 \quad m_p = 938.272088$

$m_\Lambda = 1115.683, \quad m_\mu$=105.658376

${}^3_\Lambda H = 2993.521, \quad {}^4_\Lambda H = 3933.086, \quad {}^5_\Lambda H = 4872.651,$

${}^6_\Lambda H = 5812.217, \quad {}^7_\Lambda H = 6751.782$

Table (1): Mass spectrum and binding energy of exotic muonic hypernuclei $l = 0$ in (MeV)

α_s	${}^3_\Lambda H$	${}^4_\Lambda H$	${}^5_\Lambda H$	${}^6_\Lambda H$	${}^7_\Lambda H$	$\frac{E_{bin}}{m_\mu}({}^3_\Lambda H)$
0.007	3099.1776	4038.7430	4978.3084	5917.8739	6857.4393	0.9997
0.1	3098.9144	4038.4798	4978.0452	5917.6107	6857.1761	0.9975
0.2	3098.1170	4037.6824	4977.2478	5916.8132	6856.3786	0.9899
0.3	3096.7742	4036.3396	4975.9050	5915.4705	6855.0359	0.9772
0.4	3094.8645	4034.4299	4973.9953	5913.5607	6853.1261	0.9592
0.5	3092.3549	4031.9203	4971.4857	5911.0511	6850.6166	0.9354

Table (2): Mass spectrum and binding energy of exotic muonic hypernuclei $l = 1$ in (MeV)

α_s	${}^3_\Lambda H$	${}^4_\Lambda H$	${}^5_\Lambda H$	${}^6_\Lambda H$	${}^7_\Lambda H$	$\frac{E_{bin}}{m_\mu}({}^3_\Lambda H)$
0.007	3099.1782	4038.7437	4978.3091	5917.8745	6857.4399	0.9999
0.1	3099.0467	4038.6121	4978.1776	5917.7430	6857.3084	0.9994
0.2	3098.6493	4038.2147	4977.7801	5917.3455	6856.9109	0.9975
0.3	3097.9835	4037.5489	4977.1143	5916.6797	6856.2451	0.9944
0.4	3097.0442	4036.6096	4976.1750	5915.7404	6855.3058	0.9899
0.5	3095.8238	4035.3892	4974.9546	5914.5201	6854.0855	0.9843

CONCLUSION

The exotic hyperatoms parameters have previously been studied in the low/high energy interactions with several potential models and models like the Nijmegen model, soft-core model, hard-core model, multi mesons exchange model, effective field theory chiral model, quark model, etc. This paper is devoted to determining theoretically the mass and binding energy in the framework of the ORM. An analytic expression for masses of exotic hyper systems, taking into account relativistic corrections. Without spin, interactions are derived. We have computed the masses of hyperatoms by considering the system as the nuclear core and muon. The inter hadronic interaction has been taken as electrostatically one boson exchange model. Our results have been taken from the theoretical ORM, are compared and identified with some of the experimentally or other theoretical data [7-11]. Some of our predictions bound states shown in Table1and 2 are presented as an exotic hadronic state. Maybe some of the new exotic hypernuclei-bound states very soon could be experimentally discovered in the new generation facilities, such as J-PARC, MAMI, JLab, and FAIR.

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