

## Elastic Scattering Cross Section of Pion ( $\pi^+$ ) with N, $^{16}\text{O}$ and $^{28}\text{Si}$

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The pion–nucleon and nucleus scattering at intermediate energies have been intensively studied in recent years on the basis of meson exchange theory. In this work, the optical potentials of the pion-nucleon ( $\pi^+$  N) interactions are obtained on the basis of the single-boson-exchange (SBE). Sets of boson parameters suggested by the Julich group were used to obtain the radial forms of the interacting pion nucleon optical potential  $V(r)$ , and the effective nuclear density of the target nuclei (oxygen and Silicon) was utilized to obtain the pion nucleus optical potential. The differential cross sections for ( $\pi^+$   $^{16}\text{O}$ ) and ( $\pi^+$   $^{28}\text{Si}$ ) are calculated using the first born approximation, the scattering angle in the frame of the center of mass system, within the framework of time independent Schrodinger equation, and in the  $\pi^+$  energy region  $P_{\text{lab}} < 0.5$  GeV. All results are found to be in a good agreement with the available experimental data.

**Keywords:** Strong interaction; Meson exchange; effective nuclear density; pion -nucleus optical potentials

### Introduction

The pion elastic scattering from nuclei is an important constituent in the understanding of pion nucleus interactions. Without an understanding of elastic scattering, it is not possible to investigate the physics of pion production, pion absorption and pion charge exchange. The pion-nucleon interactions are very strong, the pion mesons have zero spin and occur in three isospin states ( $\pi^+$ ,  $\pi^-$ ,  $\pi^0$ ) and their quark structure are  $u\bar{d}$ ,  $d\bar{u}$  and  $(u\bar{u} - d\bar{d})$  respectively. It is well known that the quark structures for proton and neutron are  $(uud)$  and  $(ddu)$  respectively [1, 2], and two isotopic (1/2) forms for proton and neutron Fig (1-A, 1-B).

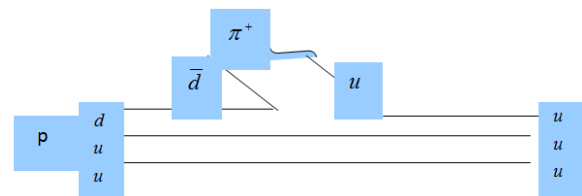


Fig. (1-A): Quark diagrams for ( $\pi^+$  P)

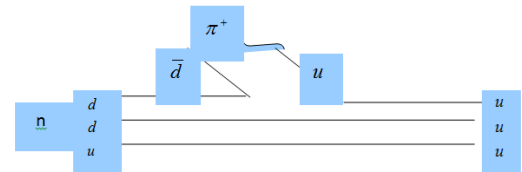


Fig. (1-B): Quark diagrams for ( $\pi^+$  n)

The isospin couplings lead us to expect that this reaction is dominated by the  $\pi^+$  proton interaction. Pion elastic scattering at 500 MeV is much less dominated by the nuclear surface, and is less sensitive to neutron and proton differences [3]. Many models for pion-nucleus scattering are built upon the impulse approximation, using a sum interaction of the mesons with free nucleons multiplied by the nuclear density [4]. The pion nucleon scattering process is based on the exchange of two mesons, one is attractive scalar meson and the other is repulsive vector meson. Therefore,  $\sigma$  and  $\rho$  exchange, which are replaced by the correlated ( $2\pi^+$ ) exchange potentials were used [5]. By this theory we construct an optical potential between the pion and nucleon, hence by using the density distribution and single folding method, the potential between the pion and nucleus have been considered [6]. The magnitude of this potential reflects the scattering of the incident particle by the individual nucleons bound in the nucleus.

The mathematical formulation of the constructed optical potential is given in section II of this study. The effectiveness of this potential by presenting the angular distributions for elastic scattering of ( $\pi^+$   $^{16}\text{O}$ ) and ( $\pi^+$   $^{28}\text{Si}$ ) at energies of 114, 163 and 240 MeV, and at range of angle 10-60 degree is investigated in section III. The results are compared to the corresponding experimental data for pion-nucleus. Section IV provides  $\pi^+$  nucleon and studied nuclei data conclusions.

The calculations of pion- nucleon and pion nucleus scattering show a good agreement with the experimental data and that the model presented here is successful for describing these types of nuclear interaction.

### Theoretical Scheme

The pion nuclear interaction is described in frame of Klein Gordon or Schrodinger equation [7, 8] using the distorted wave function in case of Born approximation. The instructive approach to the ( $\pi^+$   $N$ ) interaction potential is essentially attributed to the scalar-isoscalar  $\sigma$  and the vector-isovector  $\rho$  [5]. According to the interplay between the attractive  $\sigma$  and repulsive  $\rho$  mesons, the interaction that is responsible for the cancellation usually happen between these two fields. For pion elastic scattering from a nucleus, the transformed wave function  $\psi(r)$  which satisfies a time independent Schrodinger equation was used [8],

$$\left\{ \frac{-\hbar^2}{2\mu} \nabla^2 + U(r) \right\} \psi(r) = E_{c.m.} \psi(r), \quad (1)$$

$E_{c.m.}$  is the center-of-mass kinetic energy,

$$E_{c.m.} = \frac{(\hbar k)^2}{2\mu}, \quad (2)$$

$k$  is the wave vector, and a reduced mass  $\mu$ ,

$$\mu = \frac{m_\pi M_t}{m_\pi + M_t} \quad (3)$$

$m_\pi$  and  $M_t$  are the pion and target masses respectively, the effective bombarding pion energy  $E_L$  is related to the center-of-mass kinetic energy [9] as follows:

$$E_L = E_{c.m.} \frac{m_\pi + M_t}{M_t}, \quad (4)$$

and the  $E_L$  is the energy of the pion related to its relativistic momentum in c.m. system  $P_{c.m.}$  is as follows [10],

$$P_{c.m.} = \frac{M_t P_L}{\sqrt{(m_\pi + M_t)^2 + 2M_t(E_L - m_\pi)}}, \quad (5)$$

$P_L$  is the laboratory momentum of pion.

In equation (1) the total potential  $U(r)$  is the sum of two terms: [11],

$$U(r) = V_C(r) + U_F(r), \quad (6)$$

$V_C$  and  $U_F(r)$  are the Coulomb potential and pion nucleus potential respectively,

$$V_C(r) = \frac{z_T e^2}{R_C}, \quad (7)$$

$Z_T$  is the target atomic mass number,  $e^2 = 1.44$  MeV fm and  $R_C$  is the Coulomb radius [12].

In case of single folded the pion nucleus potential  $U_F(r)$  is related to the pion nucleon effective potential  $V_{eff(\pi^+ N)}(r)$  as [6].

$$U_F(r) = \int d r_2 \rho(r_2) V_{eff(\pi^+ N)}(r), \quad (8)$$

and  $\rho(r_2)$  is the distribution of the center of mass of the nucleons in the ground state of the  $i^{\text{th}}$  nucleus,

$$\rho(r_2) = \rho(r_n) + \rho(r_p), \quad (9)$$

while  $\rho(r_n)$  and  $\rho(r_p)$  are the neutrons and protons density in the target nucleus, the construction of the densities based on the shell model and in  $n = p$  nuclei, proton and neutron densities having the same shape. In Harmonic oscillator (HO) model [13-15],

$$\rho(r_N) = \rho_0 [1 + \alpha(r/a)^2] \exp(-(r/a)^2), \quad (10)$$

where  $\rho_0 = 0.17 \pm 0.02 \text{ fm}^{-3}$  is the nucleon density in the center of the nucleus, and the two parameter  $\alpha = 2.0$  and  $a = 1.746$  [13, 16].

Because  $\pi^+$  has a zero spin and the isospin  $\tau_\pi = 1$ , then the effective interaction potential may be written as [6],

$$V_{\text{eff}}(\pi^+ N)(r) = V_{\pi^+ N}(r) + V_{\pi^+ N}(r) \tau_\pi \cdot \tau_N, \quad (11)$$

Where  $\tau_\pi$  and  $\tau_N$  are the isospin for pion and nucleon respectively.

The potential appropriate for the elastic scattering of pion nucleon has been discussed by various researchers [17-22]. ], so in  $\pi^+ N$  interaction,

$$V_{(\pi^+ N)}(r) = V_\sigma(r) + V_\rho(r), \quad (12)$$

After long treatment of this potential as described in a previous study [23], we concluded our realistic optical potential, using the One-Boson-Exchange (OBE) model, as follows:

$$\begin{aligned} V_{(\pi^+ N)}(r) = & [ V_a + 2C_1 V_d \\ & - C_2 \frac{d^2}{dr^2} V_a \\ & - C_2 V_c \frac{d^2}{dr^2} \\ & + 2C_2 \left[ (l+1) \frac{1}{r^2} V_d \right. \\ & - 2C_2 (S \cdot l) \frac{1}{r} \frac{d}{dr} V_b \\ & \left. - 2C_2 \frac{dV_b}{dr} \frac{d}{dr} \right] \\ & + i [ 2C_3 (V_d \frac{d}{dr} + \frac{dV_d}{dr}) ], \end{aligned} \quad (13)$$

Where  $V_a(r) = -V_\sigma(r) + V_\rho(r)$ ,

$$V_b(r) = V_\sigma(r) + V_\rho(r),$$

$$V_c(r) = V_\sigma(r) + 3V_\rho(r),$$

$$V_d(r) = V_\sigma(r) - 2V_\rho(r),$$

$$C_1 = \frac{E_L^2}{8m_\pi^2 c^4},$$

$$C_2 = \frac{\hbar^2 c^2 (m_\pi c^2 + M_t c^2)^2}{8 m_\pi c^2 M_t c^2} \text{ and}$$

$$C_3 = E_L \frac{\hbar c}{8m_\pi c^2} \frac{(m_\pi + M_t) c^2}{m_\pi c^2 M_t c^2}$$

The scalar and the vector parts of single meson exchange potential are defined in earlier studies[24-27].

$$V_\sigma(r) = -\frac{g_\sigma^2}{4\pi} J_\sigma(r), V_\rho(r) = \frac{g_\rho^2}{4\pi} J_\rho(r), \quad (14)$$

Where  $\frac{g_i^2}{4\pi}$  is the coupling constant and  $J_i(r)$  is the generalized Yukawa wave function of the mesons, and  $i$  stands for the exchanged mesons  $\sigma$  and  $\rho$ .

We use the single particle energy dependant function (SPED) as Yukawa type functions [28], which is given by:

$$J_i(r) = \hbar c \left( \frac{\Lambda_i^2}{\Lambda_i^2 - u_i^2} \right) \left[ \frac{\exp(-u_i r)}{r} - \frac{\exp(-\Lambda_i r)}{r} \right] \quad (15)$$

where the parameter  $u_i = \frac{m_i c^2}{\hbar c}$  is associated with the masses of the exchanged mesons  $m_i$ ,

$\Lambda_i = \frac{\lambda_i c^2}{\hbar c}$  is associated with the cutoff masses  $\lambda_i$ .

To represent the static meson function, the suggested parameters by the Julich group [5] are presented in Table (1), and the pion, neutron and proton masses are  $m_\pi = 139.569 \text{ MeV}/c^2$ ,  $m_n = 939.565 \text{ MeV}/c^2$ , and  $m_p = 938.272 \text{ MeV}/c^2$  respectively.

**Table (1): The meson masses, coupling constants and the cut-off momentum [5]**

Meson	$m_i$ MeV/c <sup>2</sup>	$g_i^2 / \sqrt{4\pi}$	$\lambda_i$ GeV/c <sup>2</sup>
$\sigma$	550	2.385	1.85
$\rho$	769	0.84	2.4

In case of  $\pi^+ N$  system, the wave function of the pion  $\varphi_\alpha(\vec{r}_1)$  is normalized because the pion particle has zero spin, but the nucleon wave function  $\varphi_\gamma(\vec{r}_2)$  should be converted to the

$$\begin{aligned} \langle \varphi_\alpha(\vec{r}_1) \varphi_\gamma(\vec{r}_2) | &= \sum_{m_\alpha, m_\gamma, m_{s_\gamma}} (l_\alpha, 0, m_\alpha, 0 | J_\alpha, m_{J_\alpha}) \\ & (l_\gamma, \frac{1}{2}, m_\gamma, m_{s_\gamma} | J_\gamma, m_{J_\gamma}) \sum_{\lambda\mu} (l_\alpha, l_\gamma, m_\alpha, m_\gamma | \lambda\mu) \\ & \sum_{n l N L} (n_\alpha, l_\alpha, n_\gamma, l_\gamma | \lambda) | N L n l \lambda \rangle \sum_{M m} (L M m | \lambda\mu) \\ & \sum_{s m_{s_\gamma} T M_T} (0, \frac{1}{2}, 0, m_{s_\gamma} | s m_{s_\gamma}) (\frac{1}{2}, \frac{1}{2}, T_\alpha, T_\gamma | T M_T) \\ & \langle \varphi_{n l m}(\vec{r}) | \langle \varphi_{N L M}(\vec{R}) | \langle \chi_{m_{s_\gamma}}^s \hat{P}_{T_\alpha \gamma} |, \end{aligned} \quad (16)$$

the same treatment is in the Ket term

The differential cross section is given by [32]:

$$\frac{d\sigma}{d\omega}(k, \vartheta) = |f(k, \vartheta)|^2 + |f_c|^2, \quad (17)$$

where  $f(k, \vartheta)$  is the scattering amplitude and given by

$$f(k, \vartheta) = \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell+1) [\exp\{i\delta_\ell(k)\} \sin \delta_\ell(k) P_\ell(\cos \vartheta)], \quad (18)$$

where  $P_\ell(\cos \vartheta)$  is Legendre polynomial, and  $\vartheta$  is the scattering angle,  $|f_c|^2$  is the differential cross section for coulomb scattering potential [32].

$$f_c = \frac{Z_1 Z_2}{4E_1 \sin^2 \frac{1}{2} \vartheta} e^2, \quad (19)$$

In the first Born approximation the phase shift  $\delta_\ell(k)$  which is given by,

$$\tan \delta_\ell(k) = -k \langle j_\ell(kr) r | U(r) | j_\ell(kr) r \rangle \quad (20)$$

where  $j_\ell(kr)$  is a spherical Bessel function [32].

normalized function [19, 27], so the bra wave functions collected by using Clebsch-Gordan and Talmi-Moshinsky-Smirnov (GTMS) coefficients [23, 29-31], then the bra is defined by:

## Results

### The shape of the optical potential

The shape of present optical potential of pion with p, n,  $^{16}\text{O}$  and  $^{28}\text{Si}$  nuclei are plotted in the following figures.

### The elastic differential cross section

The elastic differential cross section of  $(\pi^+ - ^{16}\text{O})$  and  $(\pi^+ - ^{28}\text{Si})$  are calculated and compared with the experimental data [33, 34].

## Discussion and Conclusion

The realistic optical potential has been previously examined in case of elastic differential cross sections of Kaon meson with nucleon and some nuclei [21], and the theoretical results are fairly good. However, an examination of the elastic differential cross sections of the positive pion on p, n,  $^{16}\text{O}$  and  $^{28}\text{Si}$  at intermediate energy range of the incident pion by this potential has been conducted.

\* In fig. (2), it is clear that the maximum value of the two parts of potential in case of  $(\pi^+ n)$  are higher than that in  $(\pi^+ p)$ , this is attributed to the

effect of the total isotopic spin for system  $(\pi^+ n)$  and  $(\pi^+ p)$

\* In figs. (3, 4), in case of  $^{16}\text{O}$  or  $^{28}\text{Si}$  the absolute value of the real potential is greater than that the imaginary at the same energy, because of the effective field of the repulsive  $\rho$  meson mightier than that attractive  $\sigma$  meson. \*Also by increasing the mass number of  $^{28}\text{Si}$  to be more than that of the  $^{16}\text{O}$ , this leads to increasing the values of two parts of the optical potential. This displays the effective of nuclear medium in the scattering process.

\* In figs. (5) ( $\pi^+ ^{16}\text{O}$ ), the theoretical points are in the range of experimental results.

\* In figs. (6) ( $\pi^+ ^{28}\text{Si}$ ), it could be noticed that almost the theoretical results are in the ambit of experimental results with the exception of the points at minimum values.

\* In cases of ( $\pi^+ ^{16}\text{O}$ ) and ( $\pi^+ ^{28}\text{Si}$ ) figs. (5, 6) the values of the elastic differential cross sections increase by increasing the pion energy at the same angle.

Our theoretical calculations are in agreement, to some extent, with the experimental results, and there are no free parameters in our calculation. This means that the microscopically derived optical potential proved to be a very efficient potentia

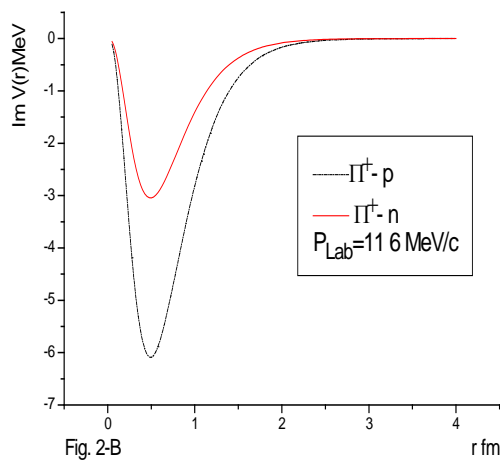


Fig. 2-B

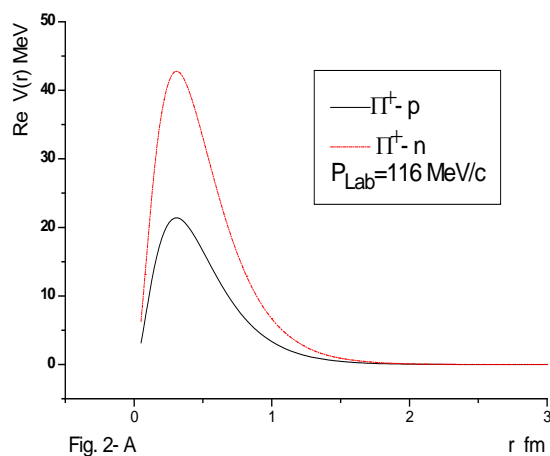


Fig. 2-A

**Figs. (2-A, B):** The real and imaginary potentials of  $\pi^+ n$  and  $p$  are plotted at pion momentum energy 116MeV/c, partial wave  $L=1$  and scattering angle in c.m system is 10.22

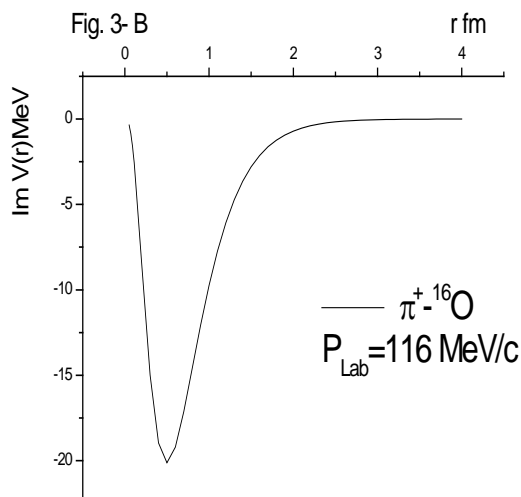


Fig. 3-B

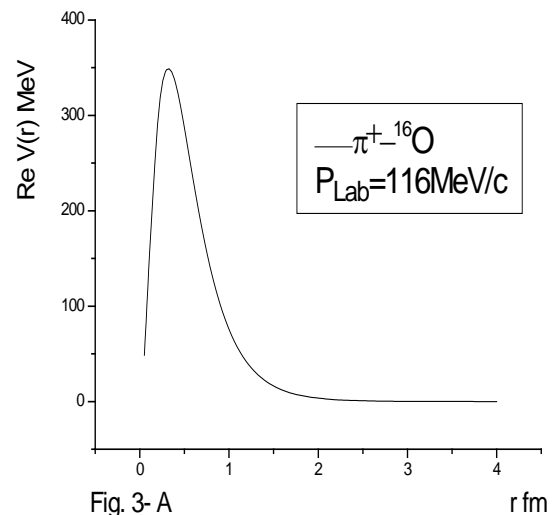
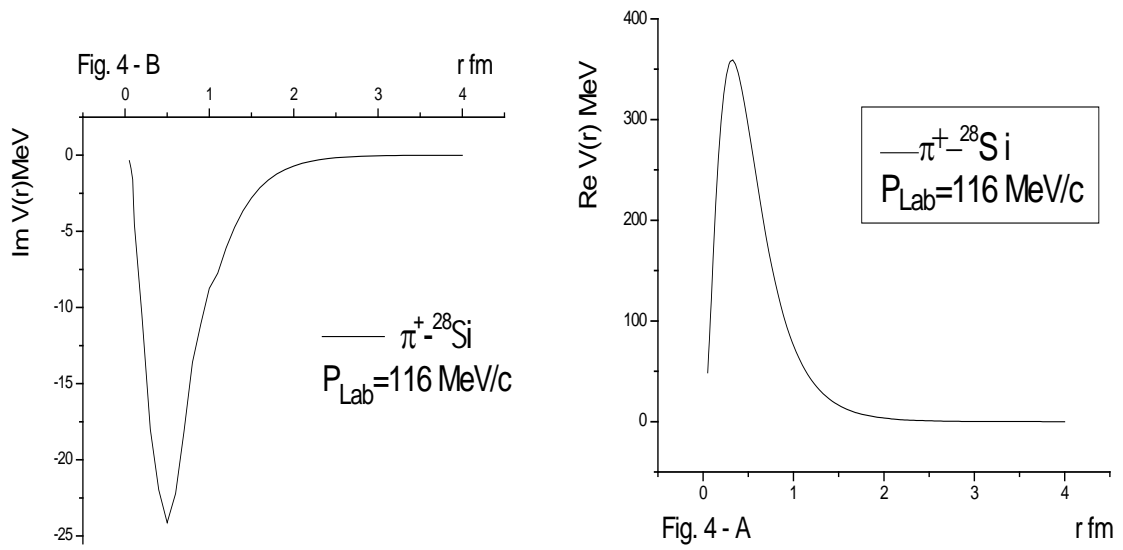
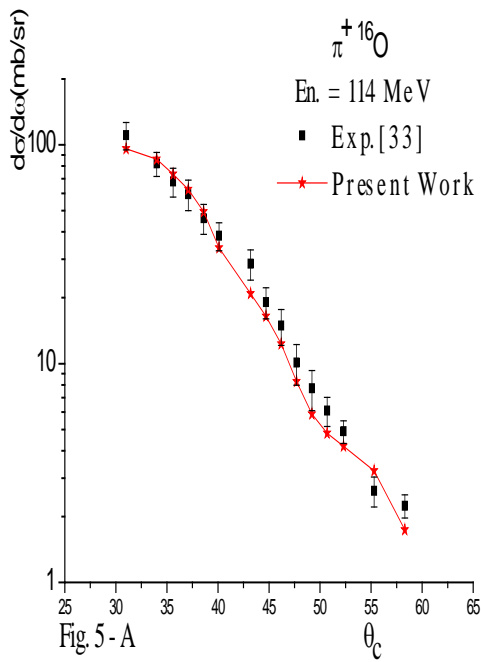


Fig. 3-A

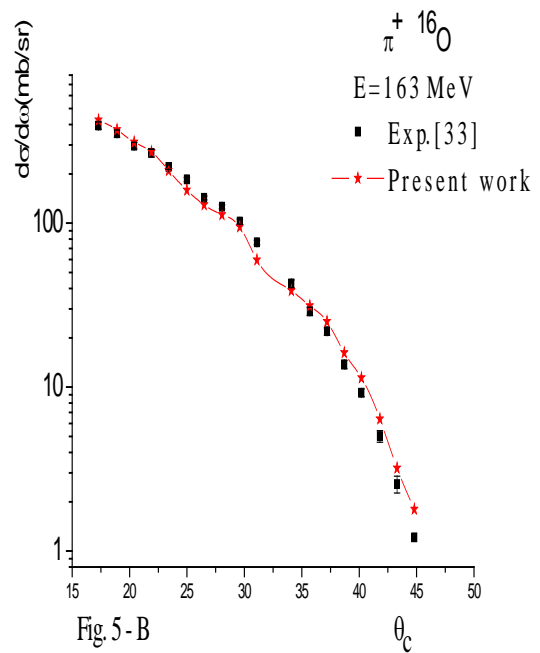
**Figs. (3-A, B):** The real and imaginary potentials of  $\pi^+ ^{16}\text{O}$  plotted at pion momentum energy 116MeV/c, partial wave  $L=1$  and scattering angle in c.m system is 10.22



**Figs. (4-A, B): The real and imaginary potentials of  $\pi^+ {}^{28}\text{Si}$  plotted at pion momentum energy 116MeV/c, partial wave  $L=1$  and scattering angle in c.m system is 10.22**



**Fig. 5-A**



**Fig. 5-B**

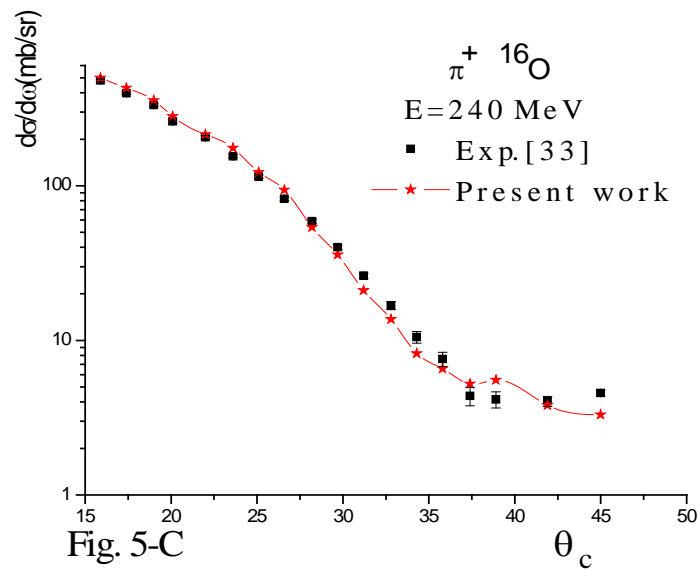
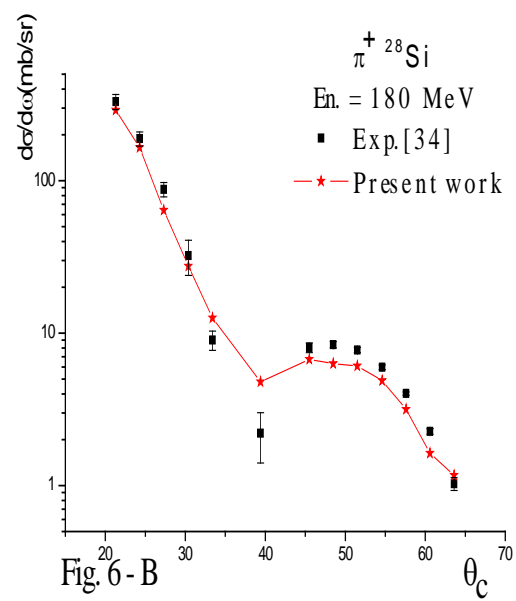
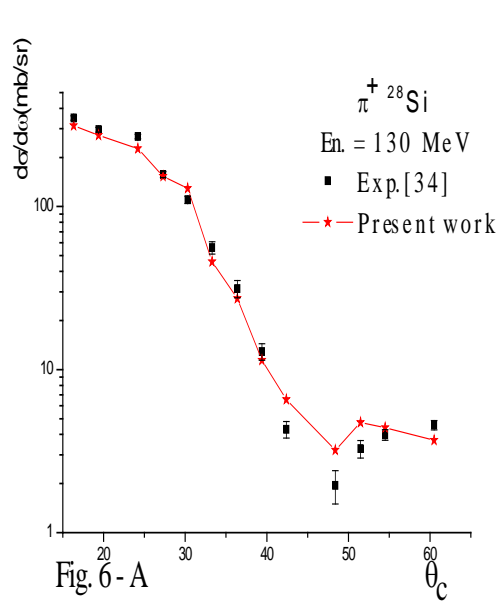


Fig. (5 - A, B, C): The elastic differential cross section for ( $\pi^+ \text{ } ^{16}\text{O}$ ) as a function of the scattering angle in the center of mass system



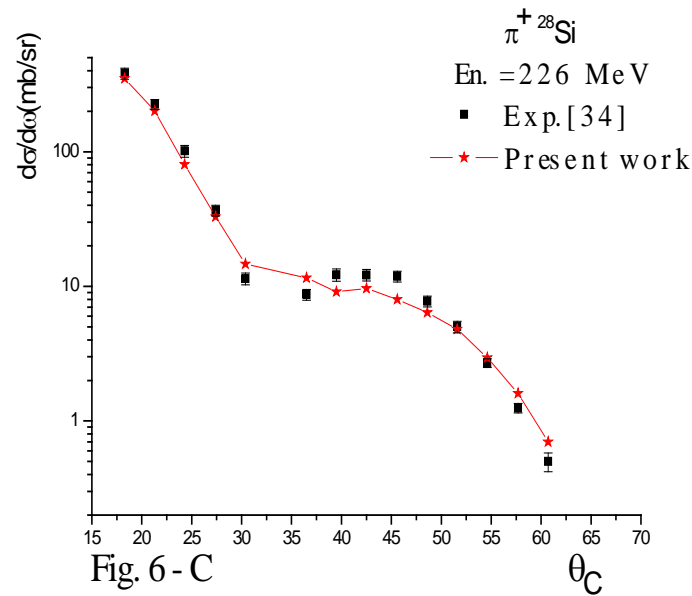


Fig. (6 - A, B, C): The elastic differential cross section for ( $\pi^+ {}^{28}\text{Si}$ ) as a function of the scattering angle in the center of mass system.

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